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Brief paper

Problems on time-varying port-controlled Hamiltonian systems: geometric structure and dissipative realization☆

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Abstract

To apply time-varying port-controlled Hamiltonian (PCH) systems to practical control designs, two basic problems should be dealt with: one is how to provide such time-varying systems a geometric structure to guarantee the completeness of representations in mathematics; and the other is how to express the practical system under consideration as a time-varying PCH system, which is called the dissipative Hamiltonian realization problem. The paper investigates the two basic problems. A suitable geometric structure for time-varying PCH systems is proposed first. Then the dissipative realization problem of time-varying nonlinear systems is investigated, and serval new methods and sufficient conditions are presented for the realization.

Keywords: Time-varying PCH system; Geometric structure; Dissipative Hamiltonian realization

1. Introduction

In recent years, time-invariant port-controlled Hamiltonian (PCH) systems have been well investigated (see, e.g., van der Schaft, 1999; Nijmeijer & van der Schaft, 1990; Maschke, Ortega, & van der Schaft, 2000; Ortega, van der Schaft, Maschke, & Escobar, 2002; Escobar, van der Schaft, & Ortega, 1999). The Hamiltonian function in a PCH system is considered as the total energy, which is the sum of potential and kinetic energies in mechanical systems, and it can play the role of Lyapunov function for the system. Because of this, based on time-invariant PCH systems, various effective controllers have been designed for many control problems (see, e.g., Shen, Ortega, Lu, Mei, & Tamura, 2000; Wang, Cheng, Li, & Ge, 2003; Xi & Cheng, 2000). However, for some practical systems the time-invariant PCH structure does not easily apply and its time-varying form is really needed. Please see the following example.

Example 1. Consider a single-machine infinite-bus power system (Lu & Sun, 1993):

$$\begin{aligned}
&\dot\delta = \omega - \omega_0,\\
&\dot\omega = \tfrac{\omega_0}{M} P_m - \tfrac{D}{M}(\omega-\omega_0) - \tfrac{\omega_0 E'_q V_s}{M x'_{d\Sigma}} \sin\delta + w_1,\\
&\dot E'_q = -\tfrac{1}{T'_d} E'_q + \tfrac{1}{T_{do}} \tfrac{x_d - x'_d}{x'_{d\Sigma}} V_s \cos\delta + \tfrac{1}{T_{d0}} u_f + w_2,
\end{aligned}$$

where w_1 and w_2 are disturbances, δ is the power angle, ω the rotor speed, E'_q the q-axis internal transient voltage, u_f the control input, and V_s the infinite-bus voltage. As for other parameters, please refer to Lu and Sun (1993). In the case that all the parameters are constant, we can use the time-invariant PCH structure to design an effective controller to attenuate the disturbances w_1 and w_2 (Xi & Cheng, 2000). But as well known, in power systems there are always uncertainties caused by load-level variations, faults, or

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changes of network structure, etc. When a parameter of the above system is affected by a time-varying signal, say, V_s is affected by a sine signal $\sin t$, the time-invariant structure is no longer valid for the system. In this case, to design an effective energy-based controller, the time-varying PCH structure is really needed.

Therefore, it is necessary to develop the theory of time-varying PCH systems for some practical control problems. Recently, time-varying PCH systems have been studied by Fujimoto and Sugie (2001a,b), Fujimoto, Sakurama, and Sugie (2003) and Cheng (2002). It is worth noticing that Fujimoto et al. (2003) set up a very important way to the trajectory tracking control of time-varying PCH systems via generalized canonical transformations, whose key idea was to preserve the structure of PCH systems under both coordinate and feedback transformations. At present, in order to apply time-varying PCH systems to practical control designs, two basic problems should be dealt with: one is how to define a geometric structure on a manifold for such systems to guarantee the completeness of representations in mathematics; and the other is how to express the practical system under consideration into a time-varying PCH system. The latter is the so-called dissipative Hamiltonian realization problem.

This paper investigates the above-mentioned two problems. First, by defining a time-varying generalized Poisson bracket, we provide a geometric structure for time-varying PCH systems. Then, we deal with the dissipative Hamiltonian realization of time-varying nonlinear systems, and propose some new methods and sufficient conditions for the realization.

The rest of the paper is organized as follows. Section 2 briefly reviews the classical Poisson structure, and Section 3 provides the geometric structure for time-varying PCH systems. In Section 4, we deal with the dissipative Hamiltonian realization problem, which is followed by the conclusion in Section 5.

2. A brief review of Poisson structure

This section briefly reviews the classical Poisson structure with Lie algebraic properties, which will motivate the next section of the paper.

In order to define a Hamiltonian system on a manifold, one should equip the manifold with a suitable geometric structure first. Let $\mathscr{M}$ be a smooth manifold and $C^\infty(\mathscr{M})$ be the set of smooth functions on $\mathscr{M}$. A Poisson bracket on $\mathscr{M}$, denoted by $\{\cdot,\cdot\}$, is a map: $C^\infty(\mathscr{M}) \times C^\infty(\mathscr{M}) \longmapsto C^\infty(\mathscr{M})$, satisfying (Ortega & Planas-Bielsa, 2004; Olver, 1993):

(i) Bilinearity:

$$\{aF + bG, H\} = a\{F, H\} + b\{G, H\},$$

$$\{F, aG + bH\} = a\{F, G\} + b\{F, H\};$$

(ii) skew-symmetry: $\{F, H\} = -\{H, F\}$;

(iii) Jacobian identity:

$$\{\{F, G\}, H\} + \{\{G, H\}, F\} + \{\{H, F\}, G\} = 0; \quad \text{and}$$

(iv) Leibniz' rule: $\{F, HG\} = \{F, H\}G + H\{F, G\}$,

where $\forall F, G, H \in C^\infty(\mathscr{M})$, $\forall a, b \in \mathbb{R}^1$. Obviously, the Poisson bracket defines a Lie algebra structure on the algebra $C^\infty(\mathscr{M})$ (Ortega & Planas-Bielsa, 2004). The pair $(\mathscr{M}, \{\cdot,\cdot\})$ is called a Poisson manifold, and the bracket defines a Poisson structure on $\mathscr{M}$.

Assume $H \in C^\infty(\mathscr{M})$ is an arbitrary smooth function. We define $X_H := \{\cdot, H\}$, which is called a Hamiltonian vector field. System $\dot{x} = X_H$ is called a Hamiltonian system defined on $\mathscr{M}$, and H is its Hamiltonian function.

It should be pointed out that the manifold $\mathscr{M}$ used here does not need to be an even-dimensional one, for the Poisson bracket defined above has dropped the property of non-degeneracy (Libermann & Marle, 1986).

In recent years, it has been well noticed that a weakening of the defining conditions of the Poisson bracket is sometimes a necessary and useful way to accommodate the description of more general dynamical systems (Ortega & Planas-Bielsa, 2004; van der Schaft, 1999; Olver, 1993). Motivated by this, in the next section we will provide a geometric structure for time-varying PCH systems.

3. Geometric structure for time-varying PCH systems

This section is to provide a geometric structure for time-varying PCH systems. First, we give the concept of time-varying generalized Poisson brackets, and then, we present the geometric structure for time-varying PCH systems.

Definition 1. Let $\mathscr{M}$ be an n-dimensional manifold and time $t \in \mathbb{R}^+ := [0, \infty)$. A time-varying generalized Poisson bracket (GPB), denoted by $\{\cdot,\cdot\}_t$, is a map: $C^\infty(\mathscr{M} \times \mathbb{R}^+) \times C^\infty(\mathscr{M} \times \mathbb{R}^+) \longmapsto C^\infty(\mathscr{M} \times \mathbb{R}^+)$, satisfying

(i) Bilinearity:

$$\begin{aligned}&\{aF(x,t) + bG(x,t), H(x,t)\}_t\\&\quad = a\{F(x,t), H(x,t)\}_t + b\{G(x,t), H(x,t)\}_t,\\&\{F(x,t), aG(x,t) + bH(x,t)\}_t\\&\quad = a\{F(x,t), G(x,t)\}_t + b\{F(x,t), H(x,t)\}_t;\end{aligned} \tag{1}$$

(ii) Leibniz' rule:

$$\begin{aligned}&\{F(x,t), G(x,t)H(x,t)\}_t\\&\quad = \{F(x,t), G(x,t)\}_t H(x,t)\\&\qquad + G(x,t)\{F(x,t), H(x,t)\}_t,\\&\{F(x,t)G(x,t), H(x,t)\}_t\\&\quad = \{F(x,t), H(x,t)\}_t G(x,t)\\&\qquad + F(x,t)\{G(x,t), H(x,t)\}_t,\end{aligned} \tag{2}$$