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Brief paper

Kalman filtering for multiple time-delay systems☆

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Abstract

This paper is to study the linear minimum variance estimation for discrete-time systems with instantaneous and l-time delayed measurements by using *re-organized innovation analysis*. A simple approach to the problem is presented in this paper. It is shown that the derived estimator involves solving $l+1$ different standard Kalman filtering with the same dimension as the original system.

Keywords: Discrete-time systems; Delayed measurements; Optimal filtering; Innovation analysis; Riccati equations

1. Introduction

The problem of estimation, which includes filtering, prediction and smoothness, has been one of the key research topics of control community since the seminal paper by Wiener (1950). The Kalman filter, which addresses the minimization of filtering error covariance (termed as H_2 estimation), emerged as a major tool of state estimation in the 1960s, see Kailath, Sayed, and Hassibi (1999), Anderson and Moore (1979) and references therein. In the past decades, the Kalman filtering has been well studied via a Riccati equation approach. It has been a classical tool in signal processing, communication and control applications. Note that the Kalman filtering formulation is only applicable to the standard systems without delays. In the time delays context, a common approach is the PDE (partial differential equation), see Kwakernaak (1967), Richard (2003), Zhang, Zhang, and Xie (2003, 2004) and references therein. This approach is usually related to solving a partial differential equation and boundary condition equations which do not have an explicit solution in general. For the case of discrete-time systems, the problem has been investigated via system augmentation and standard Kalman filtering, see Kailath et al. (1999) and Anderson and Moore (1979) or the polynomial approach (Kucera, 1979). Note that the augmented Kalman filtering approach is computationally expensive, especially when the dimension of the system is high and the measurement lags are large. On the other hand, the polynomial approach only addresses the steady-state filtering problem and it requires solving a much higher order of spectral factorization for systems with delays.

In this paper, we are concerned with the minimum mean square error (MMSE) estimation problem for the systems with instantaneous and multiple-time delayed measurement systems. Such problem has important applications in many engineering problems such as in communications and multiple-sensor fusion (Klein, 1999). Moreover, the problem studied in this paper is related to some complicated problems such as H_∞ fixed-lag smoothing (Zhang, Xie, & Soh, 2001; Zhang et al., 2004), preview control (Kojima & Ishijima, 2001) and H_∞ control with control input signal delays (Tadmor, 2002). using the *re-organized innovation sequences* (Zhang et al., 2001), we shall present a simple

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Kalman filtering formulation to the systems with l-delay measurements. It will be shown that the solution consists of l standard Kalman filters with the same dimension as the original system.

This paper is organized as follows. The problem to be addressed is stated in Section 2. Section 3 presents the main results. The comparison of the computational cost between the presented algorithm and the traditional Kalman filtering with augmentation is given in Section 4. Section 5 gives an example to show the computation procedure of the new method. The conclusions are given in Section 6.

2. Problem statement

We consider the linear discrete-time system

$$\mathbf{x}(t+1) = \Phi(t)\mathbf{x}(t) + \Gamma(t)\mathbf{u}(t), \tag{2.1}$$

where $\mathbf{x}(t) \in R^n$ is the state, and $\mathbf{u}(t) \in R^r$ is the input noise. The state is observed by l different systems with delays which are described as

$$\mathbf{y}_i(t) = H_i(t)\mathbf{x}(t_i) + \mathbf{v}_i(t), \quad i = 0, 1, \ldots, l, \tag{2.2}$$

where $t_i = t_{i-1} - d_i$, with $d_0 = 0, d_i > 0$ for $i > 0$ and $t_0 = t$. $\mathbf{y}_i(t) \in R^{p_i}$ are delayed measurements, $\mathbf{v}_i(t) \in R^{p_i}$ are the measurement noises. The initial state $\mathbf{x}(0)$, $\mathbf{u}(t)$ and $\mathbf{v}_i(t)$ $(i = 0, 1, \ldots, l)$ are uncorrelated white noises with zero means and known covariance matrices, $\mathscr{E}[\mathbf{x}(0)\mathbf{x}^{\mathrm{T}}(0)] = P_0$, $\mathscr{E}[\mathbf{u}(k)\mathbf{u}^{\mathrm{T}}(j)] = Q_u(k)\delta_{kj}$, and $\mathscr{E}[\mathbf{v}_i(k)\mathbf{v}_i^{\mathrm{T}}(j)] = Q_{v_i}(k)\delta_{kj}$ $(i = 0, 1, \ldots, l)$ respectively. In the above, $\mathscr{E}(\cdot)$ denotes the expectation.

In (2.2), $\mathbf{y}_i(t)$ means the observation of the state $\mathbf{x}(t_i)$ at time t, with delay $\sum_{k=1}^{i} d_k$. Let $\mathbf{Y}(t)$ denote the observation of system (2.1)–(2.2) at time t, then we have

$$\mathbf{Y}(t) = [\mathbf{y}_0^{\mathrm{T}}(t) \;\; \cdots \;\; \mathbf{y}_{i-1}^{\mathrm{T}}(t) \;\; 0 \;\; \cdots \;\; 0]^{\mathrm{T}},$$
$$d_1 + \cdots + d_{i-1} \leqslant t < d_1 + \cdots + d_i, \tag{2.3}$$

and for $t \geqslant d_1 + \cdots + d_l$,

$$\mathbf{Y}(t) = [\mathbf{y}_0^{\mathrm{T}}(t) \;\; \cdots \;\; \mathbf{y}_l^{\mathrm{T}}(t)]^{\mathrm{T}}. \tag{2.4}$$

For the convenience of discussion, we will consider the case of $t \geqslant d_1 + \cdots + d_l$, the other case of $t < d_1 + \cdots + d_l$ can be dealt with in the same way.

The H_2 optimal estimation problem can be stated as

Problem P. *Given the observation* $\{\{\mathbf{Y}(i)\}_{i=0}^{t}\}$, *find a linear least mean square error estimator* $\hat{\mathbf{x}}(t \mid t)$ *of* $\mathbf{x}(t)$.

Since the measurements of $\mathbf{Y}(t)$ is with different delays for the state $\mathbf{x}(t)$, the standard Kalman filtering is not applicable to the Problem P. One direct way is to convert the problem into a standard Kalman filtering estimation by augmenting the state which, however, leads to expensive computation (Anderson & Moore, 1979). The aim of this paper is to present a simple algorithm to such problem without resorting to the augmentation. The key is the re-organization of the measurements and innovation.

3. Optimal estimation with l delays

3.1. Re-organized measurements

In this subsection, the instantaneous and l-delayed measurements will be re-organized as the delay free measurements that are from $l+1$ different observation equations.

It is well known, given the measurement sequence $\{\mathbf{Y}(i)\}_{i=0}^{t}$, the optimal state estimator $\hat{\mathbf{x}}(t \mid t)$ is the projection of $\mathbf{x}(t)$ onto the linear space spanned by the measurement sequence, denoted by $\mathscr{L}\{\{\mathbf{Y}(i)\}_{i=0}^{t}\}$ (Kailath et al., 1999; Anderson & Moore, 1979). Note that the linear space $\mathscr{L}\{\{\mathbf{Y}(i)\}_{i=0}^{\mathrm{T}}\}$ is equivalent to

$$\mathscr{L}\{\mathscr{Y}_{l+1}(0), \ldots, \mathscr{Y}_{l+1}(t_l); \ldots; \mathscr{Y}_i(t_i+1), \ldots, \mathscr{Y}_i(t_{i-1}); \ldots; \mathscr{Y}_1(t_1+1), \ldots, \mathscr{Y}_1(t)\}, \tag{3.1}$$

where

$$\mathscr{Y}_i(\tau) \triangleq [\mathbf{y}_0^{\mathrm{T}}(\tau) \;\; \cdots \;\; \mathbf{y}_{i-1}^{\mathrm{T}}(\tau + \bar{d}_{i-1})]^{\mathrm{T}} \quad \text{with } \bar{d}_i = \sum_{k=1}^{i} d_k.$$

It is clear that

$$\mathscr{Y}_i(t) = \mathscr{H}_i(t)\mathbf{x}(t) + \mathscr{V}_i(t), \quad i = 1, \ldots, l+1, \tag{3.2}$$

with $\mathscr{H}_i(t) \triangleq [H_0^{\mathrm{T}}(t) \;\; \cdots \;\; H_{i-1}^{\mathrm{T}}(t+\bar{d}_{i-1})]^{\mathrm{T}}$, $\mathscr{V}_i(t) \triangleq [\mathbf{v}_0^{\mathrm{T}}(t) \;\; \cdots \;\; \mathbf{v}_{i-1}^{\mathrm{T}}(t+\bar{d}_{i-1})]^{\mathrm{T}}$, where $\mathscr{V}_i(t)$ are white noise of zero mean and covariance matrix

$$Q_{\mathscr{V}_i}(t) = diag\{Q_{v_0}(t), \ldots, Q_{v_{i-1}}(t+\bar{d}_{i-1})\},$$
$$i = 1, \ldots, l+1. \tag{3.3}$$

Note that the measurements in (2.2) are from (3.2) for $i = 1, \ldots, l+1$, which are no longer with any delay. $\{\mathscr{Y}_{l+1}(0), \ldots, \mathscr{Y}_{l+1}(t_l); \ldots; \mathscr{Y}_i(t_i+1), \ldots, \mathscr{Y}_i(t_{i-1}); \ldots; \mathscr{Y}_1(t_1+1), \ldots, \mathscr{Y}_1(t)\}$ is termed as the re-organized measurements of $\{\{\mathbf{Y}(i)\}_{i=0}^{t}\}$.

3.2. Re-organized innovation sequence

In this subsection we shall define the innovation associated with the re-organized measurements obtained in the above subsection. Now we make the following definition.

Definition 3.1. The estimator $\hat{\xi}(\tau, i)$ for $\tau > t_i + 1$ is the optimal estimation of $\xi(\tau)$ given the observation

$$\mathscr{L}\{\mathscr{Y}_{l+1}(0), \ldots, \mathscr{Y}_{l+1}(t_l); \ldots; \mathscr{Y}_i(t_i+1), \ldots, \mathscr{Y}_i(\tau-1)\}.$$

For $\tau = t_i + 1$, $\hat{\xi}(\tau, i)$ is the optimal estimation of $\xi(\tau)$ given the observation

$$\mathscr{L}\{\mathscr{Y}_{l+1}(0), \ldots, \mathscr{Y}_{l+1}(t_l); \ldots; \mathscr{Y}_{i+1}(t_{i+1}+1), \ldots, \mathscr{Y}_{i+1}(t_i)\}.$$