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Extensions to "virtual reference feedback tuning: A direct method for the design of feedback controllers"☆

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Abstract

The papers [Campi, Lecchini & Savaresi (2002). *Automatica, 38*(8), 1337–1346; (2003). *European Journal of Control*, *9*(1), 66–76] present a direct controller synthesis procedure that uses identification algorithms applied to filtered input–output plant data. This contribution discusses variations that, in some cases, may alleviate noise-induced correlation (in the open-loop case) and allow the applicability of the approach to unstable plants. Importantly, it also introduces an invalidation test step based on the available data (i.e., prior to experimental controller testing), to check if the flexibility of the controller parameterisation and the approximations involved are suitable for the design objectives or, on the contrary, the resulting closed loop may be unstable.

Keywords: System identification; Controller tuning; Adaptive control; Data-based controller design

1. Introduction

The papers (Campi, Lecchini, & Savaresi, 2002, 2003) present a controller synthesis procedure via identification (ID) techniques, directly using plant input–output data without resorting to intermediate process models. The procedure was named "*virtual reference feedback tuning*" (VRFT). This short contribution discusses simple alternatives that, in some cases, may extend the applicability of the ideas in their algorithm.[1] The interested reader is also referred to Safonov and Cabral (2001) and references therein where a related approach is described. The virtual reference approach can also be used for controller *invalidation* in supervision or adaptation tasks (Mosca & Agnoloni, 2001; Safonov & Tsao, 1997). This idea will also be addressed in this contribution. Controller ID can also be used as a controller order reduction tool (Landau, Karimi, & Constantinescu, 2001).

The structure of the contribution is based on a single discussion section, followed by examples pointing out the applicability of the presented ideas. Later, some summarising conclusions are drawn.

2. Discussion

Preliminaries and notation. Campi et al. (2002) identify a controller from input–output data (u, y) gathered from a process P, given a target closed-loop behaviour M to be attained when subject to a setpoint input r. The closed loop is depicted in figure CLS1, where $u = C(r - y)$, with a parameterised controller $C(\theta)$, $\theta \in \mathbb{R}^n$. C_0 will denote the "ideal" controller achieving the tracking behaviour M, i.e., the one with transfer function $C_0 = M(1 - M)^{-1}P^{-1}$. Depending on the parameterisation of $C(\theta)$, C_0 may not belong to the controller set $\mathscr{C}=\{C(\theta)|\theta \in \mathbb{R}^n\}$. The proposed

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[1] In the following, reference to figures and equations in Campi et al. (2002) will be denoted by prefixing with "CLS", for instance "Figure CLS1".

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controller is the one minimising cost index (CLS6), that can also be written as:

$$J_{MR}(\theta)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|W|^2|P|^{-2}|C^{-1}(\theta)-C_0^{-1}|^2}{|(PC)^{-1}+1|^2|(PC_0)^{-1}+1|^2}\,\mathrm{d}\omega, \quad (1)$$

where W is a user-defined frequency weight. The procedure evaluates a candidate controller by generating a *virtual reference*: $r(\theta)=C^{-1}(\theta)u+y$, trying to adjust it to fit a target behaviour $\bar{r}=M^{-1}y$. We will denote as *virtual tracking error* the quantity: $e=(M^{-1}-1)y$. Ideally, "perfect" control would be achieved if a parameter value θ^* were found so that $C(\theta^*)e=u$, i.e., $C(\theta^*)=C_0$. In Campi et al. (2002), it is shown that (CLS6) can be approximately minimised by posing an output-error (OE) setup with the cost index (CLS2):

$$J_{VR}^N=\frac{1}{N}\sum_{i=1}^{N}(u_L^i-C(\theta)e_L^i)^2=\|L(u-C(\theta)e)\|_{2,N}, \quad (2)$$

where u_L and e_L denote sequences, with N data points, obtained by filtering with a suitable prefilter, L, the input and virtual error sequences. The notation $\|\cdot\|_{2,N}$ has been introduced to denote the finite average of squares. The ID setup can be formulated directly on input–output data (Campi et al., 2003), as $e=(M^{-1}-1)y$. The setup proposed in Campi et al. (2002) involved a prefilter $L=M(1-M)T_u^{-1}W$, where T_u is a filter such that $|T_u|^2=\Phi_u$ (Φ_u is the power spectral density of $u(t)$), resulting in

$$J_{VR}^N=\|WM(1-M)T_u^{-1}u - C(\theta)W(1-M)^2T_u^{-1}y\|_{2,N}. \quad (3)$$

In Campi et al. (2003), analogous conditions to set up an ID experiment for the input sensitivity $C(1+PC)^{-1}$ to approach a target U are discussed. Then, by selecting different frequency weights for each of the criteria, a sort of "mixed sensitivity" approach is proposed.

Let us put forward some remarks to the procedures in Campi et al. (2002) that enhance their applicability.

Remark 1 (*full parameterisation*). In Campi et al. (2002, 2003) linearly parameterised controllers $C(\theta)=\beta^T(z)\theta$ are used, allowing for one-shot least-squares formulae. However, fully parameterised controllers $C(\theta)=(\theta_{10}+\theta_{11}z^{-1}+\cdots)/(1+\theta_{21}z^{-1}+\cdots)$ can be identified (by iterative optimisation) with widely available algorithms, such as those in Matlab System ID Toolbox. Notwithstanding, pole-zero cancellation issues appear with unstable and non-minimum phase plants, to be discussed later.

Remark 2 (*open-loop control*). If the virtual error e in (2) is replaced by the target reference $M^{-1}y$, a feedforward controller (for a stable plant) can be synthesised by direct ID. Furthermore, validation issues discussed in Section 2.1 are straightforward, as stability of the identified controller is the only requisite.

Remark 3 (*controller inverse*). An alternative setup to the one in Campi et al. (2002) may be identifying the controller inverse, using a parameterisation of $C^{-1}(\theta)$:

$$J_{IVR}^N=\|C^{-1}(\theta)Lu-L(M^{-1}-1)y\|_{2,N} \quad (4)$$

with a sensible choice of L. If the output of the process is corrupted by additive noise uncorrelated with the input u, OE algorithms will provide an unbiased estimate of θ, contrary to (2) where instrumental variables are needed. Other advantages and disadvantages of this setting with respect to the original one in Campi et al. (2002) are outlined after Remark 4.

As $Le=L(M^{-1}-1)y=L(M^{-1}-1)Pu=LC_0^{-1}u$, the minimisation of (4), asymptotically (N tending to infinity) is equivalent to minimising

$$J_{IVR}=\frac{1}{2\pi}\int_{-\pi}^{\pi}|C^{-1}(\theta)-C_0^{-1}|^2|L|^2\Phi_u\,\mathrm{d}\omega. \quad (5)$$

As the original cost index (CLS4) can be written as (1), following a line of reasoning parallel to the one yielding (CLS10) (i.e., replacing C by C_0 in the denominator of (1)) the suggested L would be

$$|L|^2=|W|^2|M|^4|P|^{-2}\Phi_u^{-1}=|W|^2|M|^4\Phi_y^{-1}. \quad (6)$$

Note that a sort of a prewhitening filter $T_y(z)$ ($|T_y|^2=\Phi_y$) needs to be identified, or approximately replaced by a high-pass filter. Inserting the above L in the original equation (4), the result can be expressed as

$$J_{IVR}^N=\|C^{-1}(\theta)WM^2T_y^{-1}u+WM(1-M)T_y^{-1}y\|_{2,N}. \quad (7)$$

Remark 4 (*unstable poles and zeros*). In the last paragraph of the example in Campi et al. (2002), the authors say that the unstable zero "does not tend to be cancelled by the controller", evidently, as the controller denominator is fixed. With unstable or non-minimum-phase plants, that may no longer be the case if a more general parameterisation, as discussed in Remark 1, is used. If the parameterisation is flexible enough, $C^{-1}(\theta)$ will tend to mimic the plant dynamics (shown by replacing y by Pu in (4)); analysing in the same way (3), $C(\theta)$ will tend to cancel the plant dynamics. To address this issue, there are three alternatives: (a) changing the parameterisation: fixing some parameters, as in the original reference (Campi et al., 2002), using reduced-order controllers, etc. (b) devising M so that the unstable or non-minimum-phase factors of P are cancelled in (2) or (4), at least approximately, and (c) using OE identification routines either in (2) or in (4) as a sole option, to obtain stable controllers with non-minimum-phase plants or to achieve stable controller inverses with unstable plants, respectively.

So, the possible advantages of ID of $C^{-1}(\theta)$ are:

(1) Tolerance to additive noise at the plant output y (with OE model structure and open-loop data).