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Robust adaptive control of nonlinear systems with unknown time delays☆

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Abstract

In this paper, robust adaptive control is presented for a class of parametric-strict-feedback nonlinear systems with unknown time delays. Using appropriate Lyapunov–Krasovskii functionals, the uncertainties of unknown time delays are compensated for. Controller singularity problems are solved by employing practical robust control and regrouping unknown parameters. By using differentiable approximation, backstepping design can be carried out for a class of nonlinear systems in strict-feedback form. It is proved that the proposed systematic backstepping design method is able to guarantee global uniform ultimate boundedness of all the signals in the closed-loop system and the tracking error is proven to converge to a small neighborhood of the origin. Simulation results are provided to show the effectiveness of the proposed approach.

Keywords: Robust adaptive control; Time-delay systems; Differentiable control

1. Introduction

In recent years, there have been tremendous efforts in adaptive control of certain class of nonlinear systems. Adaptive control has proven its great capability in compensating for linearly parameterized uncertainties. To obtain global stability, some restrictions have to be made to system nonlinearities such as matching conditions (Taylor et al., 1989), extended matching conditions (Kanellakopoulos et al., 1991), or growth conditions (Sastry & Isidori, 1989). To overcome these restrictions, a recursive and systematic backstepping design was developed in Kanellakopoulos et al. (1991). The overparametrization problem was then removed in Krstic et al. (1992) by introducing the concept of tuning function. Several adaptive approaches for nonlinear systems with triangular structures have been proposed in Seto et al. (1994) and Ge et al. (2000). Robust adaptive backstepping control has been studied for certain class of nonlinear systems whose uncertainties are not only from parametric ones but also from unknown nonlinear functions in Polycarpou and Ioannou (1996) and Pan and Basar (1998), among others. For systems that are feedback linearizable, the certainty equivalent control is usually taken the form $u(t) = [-\hat{f}(x) + v(t)]/\hat{g}(x)$, where $\hat{f}(x)$ and $\hat{g}(x)$ are estimates of $f(x)$ and $g(x)$. In this case, the assumption of $\hat{g}(x) \neq 0$ should be made to avoid the singularity problem (Yesildirek & Lewis, 1995). When $g(x)$ is referred as to virtual control coefficient with known signs, several schemes have been developed to avoid singularity problems (Wang, 1994; Spooner & Passino, 1996; Ge et al., 2002).

Practically, systems with time delays are frequently encountered (e.g., process control). Time-delayed linear systems have been intensively investigated (Kolmanovskii et al., 1999). However, the useful tools such as linear matrix inequalities (LMIs) are hard to apply to nonlinear systems with time delays. Lyapunov design has been proven to be an effective tool in controller design for nonlinear systems. One major difficulty lies in the control of time-delayed nonlinear systems is that the delays are usually not perfectly known.

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One way to ensure stability robustness with respect to this uncertainty is to employ stability criteria valid for any nonnegative value of the delays, i.e., delay-independent results. A class of quadratic Lyapunov–Krasovskii functionals (Hale, 1977) has been used earlier as checking criteria for time-delay systems' stability. The unknown time delays are the main issue to be dealt with for the extension of backstepping design to such kinds of systems. In Nguang (2000), stabilizing controller design based on the Lyapunov–Krasovskii functionals was proposed for a class of nonlinear time-delay systems with a so-called "triangular structure". However, few attempts have been made towards systems with unknown parameters or unknown nonlinear functions. In Ge et al. (2003, 2004), practical backstepping design was studied for a class of nonlinear time-delay systems in strict-feedback form by solving the problem of differentiation of the intermediate control functions at the discontinuous points in a "practical sense", i.e., setting finite values at these points, though the intermediate control functions remain not smooth at all.

Motivated by previous works on the nonlinear systems with both unknown time delays and uncertainties from unknown parameters and nonlinear functions, we present in this paper a practical robust adaptive controller for a class of unknown nonlinear systems in a parametric-strict-feedback form by employing practical yet differentiable control. Using appropriate Lyapunov–Krasovskii functionals in the Lyapunov function candidate, the uncertainties from unknown time delays are removed such that the design of the stabilizing control law is free from these uncertainties. In this way, the iterative backstepping design procedure can be carried out directly. A novel smooth approximation is introduced to solve the differentiability problem for the intermediate control functions. Time-varying control gains rather than fixed gains are chosen to guarantee the boundedness of all the signals in closed-loop system. The global uniform ultimate boundedness (GUUB) of the signals in the closed-loop system is achieved and the output of the systems is proven to converge to a small neighborhood of the desired trajectory.

To the best of our knowledge, there is little work dealing with such a kind of systems in the literature at present stage. The proposed method expands the class of nonlinear systems that can be handled using adaptive control techniques. The main contributions of the paper lie in: (i) the employment of robust adaptive backstepping controller design for a class of unknown nonlinear time-delay systems in parametric-strict-feedback form, in which the unknown time delays are compensated for by using appropriate Lyapunov–Krasovskii functionals; (ii) the introduction of differentiable practical control in solving the controller singularity problem so that it can be carried out in backstepping design and guarantee the tracking error being confined in a compact domain of attraction; and (iii) the elegant re-grouping of unknown parameters, by which the controller singularity problem is effectively avoided, and the lumping of unknown parameter vectors as scalars, by which the number of parameters being estimated as well as the order and complexity of the controllers, are dramatically reduced.

2. Problem formulation and preliminaries

Consider a class of single-input-single-output (SISO) nonlinear time-delay systems

$$\begin{aligned}\dot{x}_i(t) &= g_i x_{i+1}(t) + f_i(\bar{x}_i(t)) \\ &\quad + h_i(\bar{x}_i(t-\tau_i)), \quad 1 \leqslant i \leqslant n-1, \\ \dot{x}_n(t) &= g_n u(t) + f_n(x(t)) + h_i(x(t-\tau_n)), \\ y(t) &= x_1(t), \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}}$, $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$, $u \in R$, $y \in R$ are the state variables, system input and output, respectively, $f_i(\cdot)$ and $h_i(\cdot)$ are unknown smooth functions, g_i are unknown constants, and τ_i are unknown time delays of the states, $i = 1, \ldots, n$. The control objective is to design an adaptive controller for system (1) such that the output $y(t)$ follows a desired reference signal $y_d(t)$, while all signals in the closed-loop system are bounded. Define the desired trajectory $\bar{x}_{d(i+1)} = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^{\mathrm{T}}$, $i = 1, \ldots, n-1$, which is a vector of y_d up to its ith time derivative $y_d^{(i)}$. We have the following assumptions for the system functions, unknown time delays and reference signals.

Assumption 1. The signs of g_i are known, and there exist constants $g_{\max} \geqslant g_{\min} > 0$ such that $g_{\min} \leqslant |g_i| \leqslant g_{\max}$.

The above assumption implies that unknown constants g_i are strictly either positive or negative. Without losing generality, we shall only consider the case when $g_i > 0$. It should be emphasized that the bounds $g_{\min}$ and $g_{\max}$ are only required for analytical purposes, their true values are not necessarily known since they are not used for controller design.

Assumption 2. The unknown functions $f_i(\cdot)$ and $h_i(\cdot)$ can be expressed as

$$f_i(\bar{x}_i(t)) = \theta_{fi}^{\mathrm{T}} F_i(\bar{x}_i(t)) + \delta_{fi}(\bar{x}_i(t)),$$

$$h_i(\bar{x}_i(t)) = \theta_{hi}^{\mathrm{T}} H_i(\bar{x}_i(t)) + \delta_{hi}(\bar{x}_i(t)),$$

where $F_i(\cdot)$, $H_i(\cdot)$ are known smooth function vectors, $\theta_{fi} \in R^{n_i}$, $\theta_{hi} \in R^{m_i}$ are unknown constant parameter vectors, n_i, m_i are positive integers, $\delta_{fi}(\cdot)$, $\delta_{hi}(\cdot)$ are unknown smooth functions, which satisfy the so-called triangular bounds conditions

$$|\delta_{fi}(\bar{x}_i(t))| \leqslant c_{fi}\phi_i(\bar{x}_i(t)),$$

$$|\delta_{hi}(\bar{x}_i(t))| \leqslant c_{hi}\psi_i(\bar{x}_i(t)),$$

where c_{fi}, c_{hi} are constant parameters, which are not necessarily known, and $\phi_i(\cdot)$, $\psi_i(\cdot)$ are known nonnegative smooth functions.