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Digital repetitive plug-in controller for odd-harmonic periodic references and disturbances☆

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Abstract

This work develops a digital repetitive plug-in controller for odd-harmonic discrete-time periodic references and disturbances. The controller presents a novel structure and it has a lower data memory occupation than the usual repetitive controllers because it takes advantage of the particular characteristics of the signals to track or attenuate. A sufficient criterion of stability, several hints for its practical application and an example are also included in the work.

Keywords: Sampled-data control; Internal model principle; Repetitive control; Tracking control; Periodic disturbances

1. Introduction

A common strategy in control systems that have periodic reference and disturbance signals is the Internal Model Principle (IMP) (Francis & Wonham, 1976). According to this principle, in order to achieve zero tracking error in steady state it is necessary and sufficient to include the generators of the reference signal and/or the disturbance signal in the loop, either in the plant itself or in the controller. The specific application of the IMP to the class of periodic signals is usually known as repetitive control (Yamamoto, 1993; Hillerström & Walgama, 1996).

The concept of repetitive control has been largely used in different control areas such as CD and disk arm actuators (Chew & Tomizuka, 1990a), robotics (Yamada, Riadh, & Funahashi, 1999), electro-hydraulics (Kim & Tsao, 2000), electronic rectifiers (Zhou, Wang, & Xu, 2000), pulse-width modulated (PWM) inverters (Zhou, Wang, & Low, 2000; Zhou & Wang, 2001), and current harmonics active filters (Griñó, Costa-Castelló, & Fossas, 2003).

When repetitive control is used in discrete time, a sampling period (T_s) which is a submultiple of the periodic signal period (T_p) must be chosen. Under this assumption the transfer function that should be included inside the loop is

$$G_r(z) = \frac{z^{-N}}{1 - z^{-N}} = \frac{1}{z^N - 1}, \qquad (1)$$

where $N = T_p/T_s \in \mathbb{N}$. It is important to note that the discrete-time implementations can only keep under control those harmonics whose frequencies are below the half of the sampling frequency, i.e. π/T_s.

It is usual, especially in power electronics systems, that the reference and disturbance signals appearing in the system have only odd-harmonic frequencies; if the traditional repetitive control approach is used in these situations infinite gain is introduced at even harmonic frequencies too. This fact reduces system robustness, and does not improve system performance. In this work, a new approach to repetitive control for this kind of signals is presented. The proposed scheme only introduces infinite gain at odd-harmonic frequencies; in other words it only introduces infinite gain

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where it is needed. Moreover, sometimes the power electronics systems include voltage sensing with transformers and then the open-loop transfer function has pure derivatives. So any controller with integral action, like PID controllers or the traditional repetitive controllers, must be precluded in order to obtain an internally stable closed-loop system.

2. Odd-harmonic discrete-time plug-in repetitive controller

2.1. Odd-harmonic discrete-time periodic signal generators

As is known, a discrete-time signal $x(n) \in \mathbb{C}$ is periodic with period $N \in \mathbb{N}$ if and only if $x(n+N) = x(n),\ \forall n \in \mathbb{Z}$, where the least value of N that verifies the equation is called the fundamental period of $x(n)$. Also, the discrete-time periodic signal $x(n)$ can be developed, or represented, in Fourier series as follows:

$$x(n) = \sum_{k=0}^{N-1} c_k \mathrm{e}^{j2\pi kn/N}, \tag{2}$$

where the coefficients of the Fourier series representation $\{c_k\}$,

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\mathrm{e}^{-\mathrm{j}2\pi kn/N} \in \mathbb{C},$$

form a periodic sequence with fundamental period N when extended outside of the range $k = 0, 1, \ldots, N-1$.

Definition 1. A discrete-time periodic signal $x(n)$, with fundamental even period N, is called an odd-harmonic periodic signal if its Fourier series coefficients c_k of even index ($k \bmod 2 = 0$)(including $k = 0$) are zero.

This property has its counterpart in the time domain as the following proposition states.

Proposition 2. *A discrete-time periodic signal $x(n)$, with fundamental even period N, is an odd-harmonic periodic signal if and only if $x(n + N/2) = -x(n)$.*

Proof. Straightforward from the Fourier series representation in Eq. (2). □

The following proposition states the structure of the odd-harmonic discrete-time periodic signal generators used in the work from now on.

Proposition 3. *The z-transform of a discrete-time odd-harmonic periodic signal $x(n)$, with fundamental even period N, is*

$$X(z) \triangleq \mathfrak{z}\{x(n)\} = \frac{z^{N/2}}{z^{N/2}+1} X_1(z), \tag{3}$$

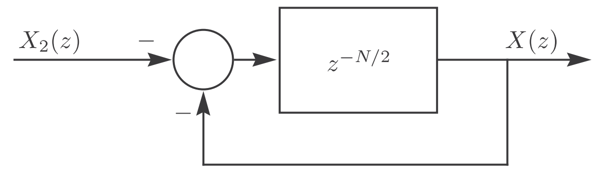

Fig. 1. Repetitive loop for odd-harmonic discrete-time periodic signals.

where $X_1(z) \triangleq \mathfrak{z}\{x_1(n)\}$ and

$$x_1(n) = \begin{cases} x(n), & 0 \leqslant n \leqslant \frac{N}{2} - 1, \\ 0, & elsewhere. \end{cases}$$

Proof. $x_1(n)$ is, apart from the sign, a semi-period of $x(n)$, so $x(n)$ can be constructed from $x_1(n)$ as a sum of shifted copies of $x_1(n)$, that is

$$x(n) = \sum_{k=0}^{\infty} x_1\left(n - k\frac{N}{2}\right)(-1)^k$$

and, taking the z-transform, results in

$$X(z) = \sum_{k=0}^{\infty} (-1)^k z^{-kN/2} X_1(z) = \frac{z^N/2}{z^{N/2}+1} X_1(z). \qquad \square$$

Then, the discrete-time periodic generator can also be seen, with a semi-period delay, as the following transfer function equation (see Fig. 1):

$$X(z) = -\frac{z^{-N/2}}{1+z^{-N/2}} X_2(z) = -\frac{1}{z^{N/2}+1} X_2(z), \tag{4}$$

where $x_2(n) = -x_1(n - N/2)$ and, equivalently, $X_2(z) = -X_1(z)\, z^{-N/2}$. That is to say, the system of Fig. 1 with initial conditions $x_2(n)$.

The odd-harmonic periodic signal generator in Eq. (4) has its poles in $z = \mathrm{e}^{j(2k+1)2\pi/N},\ k = 0, 1, \ldots, (N/2) - 1$. Then, as N is the period of the signal, the frequencies associated to the poles $\omega_k = (2k+1)2\pi/N$ correspond to the frequencies of all the odd harmonics of the Fourier series development of the periodic signal. So, the transfer function of Eq. (4) will have infinite gain, because its poles have modulus 1, in these frequencies and, moreover, if it is incorporated in the open loop transfer function of a system then it will achieve perfect asymptotic tracking or disturbance rejection for this class of periodic signals.

Compared to the traditional characteristic equation ($z^N - 1 = 0$) in discrete-time repetitive control, this approach means that all the even harmonic poles have disappeared from the pole map. It is worth noting that the pole in $\omega = 0$ ($k = 0$), as an even harmonic frequency pole, also disappears and, then, there is no pure integrator introduced by the repetitive controller in this approach.