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Control Engineering Practice 13 (2005) 659–666
CONTROL ENGINEERING PRACTICE
www.elsevier.com/locate/conengprac

An improved linear fractional model for robustness analysis of a winding system ☆

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Received 24 February 2003; received in revised form 24 May 2004; accepted 26 May 2004

Available online 8 July 2004

Abstract

Web winding systems exhibit nonlinear dynamics and their behaviors vary with respect to several quantities such as elasticity modulus and end roll radii. Being able to guarantee that the system remains stable during the whole winding process when parameters vary appears to be crucial. μ-analysis is a classical technique for evaluating robustness of parameter-dependent systems and is used herein. It requires the model to be under the ad hoc form of a linear fractional representation. Following the physics equations, the different steps are detailed until to come to the ad hoc model including parameters with independent variations. Notably, the nonlinear dependence between the winder and unwinder radii is considered. Numerical results are given in the case where the system is driven by a H_∞ controller.

Keywords: LFR; Winding system; μ-analysis; Robustness analysis; Modelling

1. Introduction

Web winding and transporting systems are commonly used in the industry with metal, paper, fabric, etc. Controllers are included in these systems in order to control the tension and the speed of the web. Decentralized PID controllers are generally used. Recently, multivariable H_∞ control techniques were proposed for these systems and showed good results for systems including 2 or 3 actuators (Koç, Knittel, & de Mathelin, 2002). For systems with a higher number of actuators, decentralized techniques can be used (Knittel, Gigan, Laroche, & de Mathelin, 2002). Those techniques rely on a linear model obtained for a setting point. As the models are nonlinear and as the radii vary during the winding process, the behavior of the system varies and moves from the nominal behavior. A robustness analysis then appears to be a necessary step in order to validate the controller.

The system dynamics include smooth nonlinearities. Nevertheless, as parameters and state variables vary slowly during winding processes, the behavior around a setting point is well reproduced by the linearized model. Thus the robustness analysis can be turned into the problem of checking the stability of a linear parameter-varying (LPV) model with slowly varying parameters. This issue, the so called μ-analysis based on the structured singular value, appears to be a convenient method and efficient algorithms (Doyle, 1982; Packard & Doyle, 1993) are available. In application to winding systems, this method was first used in order to check robustness to the elasticity modulus of a three motor winding system (Koç, Knittel, de Mathelin, & Abba, 2000). In (Laroche, Knittel, Koç, & de Mathelin, 2001), the method was extended for accounting variations of speed, tension and radii. The radii of the winder and unwinder were then considered as independent quantities. However they are not, as their square sum remains constant, leading to pessimism in the robustness evaluation. The aim of this paper is to show a complete

☆A preliminary version of this work was published in Laroche and Knittel (2003).

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0967-0661/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.conengprac.2004.05.008

robustness evaluation procedure with great attention to the development of the linear fractional representation (LFR). It notably includes removing the interdependence of several quantities, the radii in the present case. The procedure presented is general and can be applied to the robustness analysis of any kind of system. Thanks to a toolbox dedicated to LFR systems (Magni, 2001), the development of the models is highly automated. The LFR model is then used to check robustness of a 3-motor system controlled by a H_∞ multivariable controller. Two methods are considered for computing μ bounds: a method based on the root locus and the classical μ upper bound with frequency gridding.

The paper is organized as follows. In Section 2, the winding system, its physical models and the H_∞ controller are presented. In Section 3, the LFR model, including the interdependence between winder and unwinder radii, is developed. This representation is a prerequisite for μ-analysis. Comparisons between the nonlinear and the linear models are done in order to validate the LFR model. Section 4 is dedicated to robustness evaluation; methods for computing bounds on μ are presented and results are included.

2. Presentation of the system

2.1. General description

The winding process considered in the paper is represented in Fig. 1. It includes 5 rolls numbered 1–5 from left to right. Three rolls (numbered 1, 3 and 5) are motored by torque controlled DC motors: the unwinder (torque reference u_1), the tractor (torque reference u_2) and the winder (torque reference u_3). The roll radii are denoted R_k, the tension and speed of the web in its different parts are T_k and V_k, while the rotational speeds of the rolls are denoted Ω_k. Two tension sensors are located on rolls 2 and 4; they provide the mean of the tensions at the left and at the right of the roll. The motor torque reference signals u_k are computed in order to control the web speed and the two measured tensions.

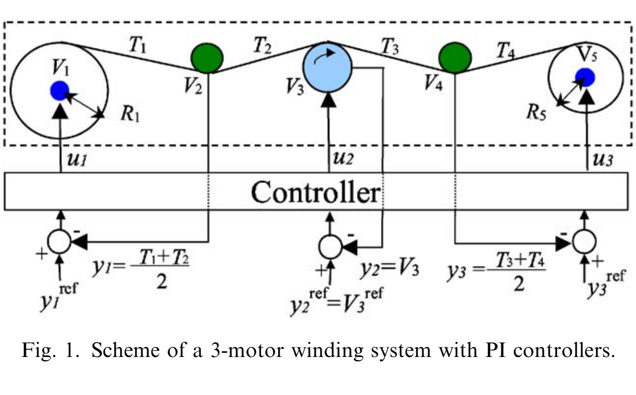

Fig. 1. Scheme of a 3-motor winding system with PI controllers.

2.2. Physics equations

The model of web transport systems is briefly given in this paragraph; the reader may refer to (Koç, 2000; Koç et al., 2002) for complementary information. This model is based on the laws of physics.

- Hooke's law allows for web elasticity.
- Coulomb's law explains the contact between the web and the roll, including friction.
- The mass conservation law allows for the coupling between the speed and the tension of the web.
- The second fundamental relation of dynamics accounts for variations of the rotating speeds.

With adequate hypotheses, dynamic equations can be found for the tension of each part of the web and for the speed of each roll. For instance, equations giving the dynamics of T_1 and V_3 are

$$L_1 \frac{\mathrm{d}T_1}{\mathrm{d}t} = V_2(ES + T_2) - V_1 \frac{(ES + T_2)^2}{ES + T_1},$$
$$\frac{\mathrm{d}}{\mathrm{d}t}(J_3\Omega_3) = K_2 U_2 + R_3(T_2 - T_3) - f_3(\Omega_3), \tag{1}$$

where E is the elasticity modulus of the web, S is its section, L_1 is the length between the two first rolls, J_3 is the inertia of the third roll, K_2 is the torque-tension ratio of the second motor and f_3 is the friction torque depending on rotation speed Ω_3.

The mass conservation law also accounts for variations of the winder and unwinder radii

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = -\frac{e}{2\pi}\Omega_1,$$
$$\frac{\mathrm{d}R_5}{\mathrm{d}t} = \frac{e}{2\pi}\Omega_5, \tag{2}$$

where e is the web thickness. The inertias of the winding and unwinding rolls vary in an affine manner with the fourth power of the radii:

$$J_k = J_{k0} + \lambda_k R_k^4. \tag{3}$$

2.3. Identification

The parameters and the friction maps need to be identified. This section briefly recalls the results presented in Koç et al. (2002) and in Koç (2000) that are necessary for the robustness analysis.

2.3.1. Friction

For rollers including motors, friction maps $f_k(\Omega_k)$ were identified independently in an experiment where the web was removed in order to disconnect the motor from the rest of the setup. At steady-state, one has $K_3 U_3 = f_3(\Omega_3)$, allowing to identify f_3 assuming K_3 known. A polynomial development of order 3 was