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Gain scheduling control of variable-speed wind energy conversion systems using quasi-LPV models

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Abstract

The paper deals with the control of variable-speed wind energy conversion systems (WECS) in the context of linear parameter varying (LPV) systems, a recent formulation of the classic gain scheduling technique. The LPV approach is specially useful in variable-speed WECS control, which is characterized by nonlinear dynamic behavior and opposite objectives. In particular, the following objectives are considered: conversion efficiency maximization, safe operation, resonant modes damping, and robust stability. The proposed LPV controller is compared with a fixed controller that also takes into account the nonlinear behavior of the wind turbine.

Keywords: Linear parameter varying systems; Gain scheduling; Wind energy conversion systems; Wind turbines

1. Introduction

Nowadays, variable-speed wind energy conversion systems (WECS) are receiving considerable interest because they are able to maximize the energy capture and to reduce the aerodynamic load for a wide range of wind speeds. In variable-speed WECS, an electronic converter uncouples the rotational speed from the grid frequency, allowing the wind turbine to work at optimal operating conditions at different wind speeds (Leithead & Connor, 2000; Muljadi, Pierce, & Migliore, 2000; Thresher & Dodge, 1998). Also, it is known that WECS present nonlinear dynamic behavior and lightly damped resonant modes. When the frequency range of the disturbances matches one of the resonant modes, the life of the turbine components is reduced, and the generated power quality is deteriorated (Freris, 1990; Novak, Ekelund, Jovik, & Schmidtbauer, 1995). Typical objectives are: to maximize energy capture in low wind speeds, to maintain the generated power and the rotational turbine speed within safe limits during high wind speeds, and to avoid lightly damped resonant modes in the closed loop system (Leithead & Connor, 2000; Novak et al., 1995).

On the other hand, control techniques based on gain scheduling concepts are extensively used in practical applications. The classic gain scheduling approach consists in designing linear controllers for several operating points and then applying an interpolation strategy to obtain a global control. Consequently, powerful tools for linear systems can be applied to nonlinear plants. In spite of the numerous applications, there was not a formal framework until the beginning of the nineties (Rugh, 1991; Shamma & Athans, 1990). This framework gives heuristic rules to ensure global stability, but it does not provide a systematic design procedure. Later, Shamma & Athans (1991) introduce the linear parameter varying (LPV) systems. In this context, the synthesis problem can be formulated as a convex optimization problem with linear matrix inequality (LMI) constraints wherein the controller is considered as a simple entity without the classical interpolations drawbacks (Packard, 1994; Apkarian & Gahinet, 1995; Becker & Packard, 1994; Apkarian, Gahinet, & Becker, 1995).

This paper deals with the modelling and control of variable-speed WECS using the LPV gain scheduling approach. Analogously to linear optimal control, it is

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possible to design a controller that considers the nonlinear nature of wind turbines, aims to balance opposed objectives, and ensures stability with model uncertainty.

This paper is structured as follows, Section 2, presents a brief summary of LPV gain scheduling techniques. In Section 3, the dynamic equations of WECS are deduced. Then, in Section 4, the problem specifications are discussed. Finally, the proposed LPV gain scheduling controller is presented in Section 5.

2. LPV Gain scheduling

Linear parameter varying (LPV) systems can be considered as a particular case of linear time varying (LTV) systems where the matrices of the state model are continuous and fixed functions of some varying parameter vector $\boldsymbol{\theta}(t) \in \mathbb{R}^s$. That is

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{B}(\boldsymbol{\theta}(t))\mathbf{w}(t),\\ \mathbf{z}(t) &= \mathbf{C}(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{D}(\boldsymbol{\theta}(t))\mathbf{w}(t),\end{aligned} \tag{1}$$

where $\mathbf{x}$ is the state ($\mathbf{x} \in \mathbb{R}^n$), $\mathbf{w}$ is the input, and $\mathbf{z}$ is the output. The parameter vector $\boldsymbol{\theta}(t)$ is not known a priori, but it is assumed in a bounded set $\Theta \subset \mathbb{R}^s$.

If there exits a symmetric positive definite matrix $\mathbf{P} \in \mathbb{R}^{n\times n}$ such that

$$\begin{bmatrix} \mathbf{A}(\boldsymbol{\theta})^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}(\boldsymbol{\theta}) & \mathbf{P}\mathbf{B}(\boldsymbol{\theta}) & \mathbf{C}(\boldsymbol{\theta})^{\mathrm{T}} \\ \mathbf{B}(\boldsymbol{\theta})^{\mathrm{T}}\mathbf{P} & -\gamma\mathbf{I} & \mathbf{D}(\boldsymbol{\theta})^{\mathrm{T}} \\ \mathbf{C}(\boldsymbol{\theta}) & \mathbf{D}(\boldsymbol{\theta}) & -\gamma\mathbf{I} \end{bmatrix} < 0 \tag{2}$$

for any possible trajectory $\boldsymbol{\theta}(t)$, the system (1) is exponentially stable, and it can be assured that $\|\mathbf{z}\|_2 \leqslant \gamma\|\mathbf{w}\|_2$ $\forall \boldsymbol{\theta}(t) \in \Theta$ and $\gamma > 0, \gamma \in \mathbb{R}$ (Apkarian et al., 1995; Becker & Packard, 1994).

Given the open loop system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{B}_1(\boldsymbol{\theta}(t))\mathbf{w}(t) + \mathbf{B}_2(\boldsymbol{\theta}(t))\mathbf{u}(t),\\ \mathbf{z}(t) &= \mathbf{C}_1(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{D}_{11}(\boldsymbol{\theta}(t))\mathbf{w}(t) + \mathbf{D}_{12}(\boldsymbol{\theta}(t))\mathbf{u}(t),\\ \mathbf{y}(t) &= \mathbf{C}_2(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{D}_{21}(\boldsymbol{\theta}(t))\mathbf{w}(t)\end{aligned} \tag{3}$$

with the control input $\mathbf{u}$ and the measured output $\mathbf{y}$, the LPV gain scheduling synthesis problem consists in finding a controller

$$\begin{aligned}\dot{\mathbf{x}}_k(t) &= \mathbf{A}_k(\boldsymbol{\theta}(t))\mathbf{x}_k(t) + \mathbf{B}_k(\boldsymbol{\theta}(t))\mathbf{y}(t),\\ \mathbf{u}(t) &= \mathbf{C}_k(\boldsymbol{\theta}(t))\mathbf{x}_k(t) + \mathbf{D}_k(\boldsymbol{\theta}(t))\mathbf{y}(t)\end{aligned} \tag{4}$$

such that the closed loop system satisfies (2). Notice that although the parameter vector $\boldsymbol{\theta}(t)$ must be measured in real-time, for the controller design only the bounded set Θ is required.

This synthesis problem can be formulated as a convex optimization problem with LMI constraints (Apkarian et al., 1995; Becker & Packard, 1994; Apkarian & Gahinet, 1995; Packard, 1994). Hence, a complete and systematic solution using the efficient interior point algorithms is achieved (Gahinet, Nemirovskii, Laud, & Chilali, 1994). In this context, Apkarian and Adams (1998) present the basic characterization to incorporate into the synthesis problem multiple specifications such as $\mathscr{H}_2/\mathscr{H}_\infty$ constraints and pole clustering. In the same work, the authors consider parameter-dependent scalings to exploit the structural information on the operator $\mathbf{w} \to \mathbf{z}$.

The mathematical formulation required to synthesize the proposed controller is summarized as follows. The plant (3) is considered with $\mathbf{w}$ and $\mathbf{z}$ subject to

$$[\mathbf{w}_1(t), \ldots, \mathbf{w}_m(t)]^{\mathrm{T}} = \boldsymbol{\Delta}(t)[\mathbf{z}_1(t), \ldots, \mathbf{z}_m(t)]^{\mathrm{T}},$$

where the operator $\Delta(t)$ has the following structure:

$$\boldsymbol{\Delta} := \mathrm{diag}(\boldsymbol{\Delta}_1(t), \ldots, \boldsymbol{\Delta}_m(t)) \quad \text{with} \quad \sigma_{max}(\boldsymbol{\Delta}(t)) \leqslant 1/\gamma, \quad \forall t \geqslant 0.$$

Also, the following associated set of parameter-dependent scalings is defined:

$$\mathscr{S}_{\boldsymbol{\Delta}} := \{\mathbf{S}: \ \mathbf{S} > 0, \mathbf{S}\boldsymbol{\Delta}(t) = \boldsymbol{\Delta}(t)\mathbf{S}, \ \forall t \geqslant 0\}.$$

Assuming that the parameter dependence of the plant (3) is affine, $\mathbf{B}_2$, $\mathbf{C}_2$, $\mathbf{D}_{12}$, $\mathbf{D}_{21}$ are constant and Θ is a polytope with vertices $\boldsymbol{\theta}_i$, then, the controller can be obtained by solving the following set of LMIs

$$\begin{bmatrix} \mathbf{X}\mathbf{A}_i + \hat{\mathbf{B}}_{k_i}\mathbf{C}_2 + (\bigstar) & \bigstar & \bigstar & \bigstar \\ \hat{\mathbf{A}}_{k_i}^{\mathrm{T}} + \mathbf{A}_i + \mathbf{B}_2\mathbf{D}_{k_i}\mathbf{C}_2 & \mathbf{A}_i\mathbf{Y} + \mathbf{B}_2\hat{\mathbf{C}}_{k_i} + (\bigstar) & \bigstar & \bigstar \\ \mathbf{S}_i^{-1}(\mathbf{X}\mathbf{B}_{1_i} + \hat{\mathbf{B}}_{k_i}\mathbf{D}_{21})^{\mathrm{T}} & \mathbf{S}_i^{-1}(\mathbf{B}_{1_i} + \mathbf{B}_2\mathbf{D}_{k_i}\mathbf{D}_{21})^{\mathrm{T}} & -\gamma\mathbf{S}_i^{-1} & \bigstar \\ \mathbf{C}_{1_i} + \mathbf{D}_{12}\mathbf{D}_{k_i}\mathbf{C}_2 & \mathbf{C}_{1_i}\mathbf{Y} + \mathbf{D}_{12}\hat{\mathbf{C}}_{k_i} & (\mathbf{D}_{11} + \mathbf{D}_{12}\mathbf{D}_{k_i}\mathbf{D}_{21})\mathbf{S}_i^{-1} & -\gamma\mathbf{S}_i^{-1} \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix} > 0, \tag{6}$$

where $\mathbf{X}$, $\mathbf{Y}$, $\hat{\mathbf{A}}_{k_i}$, $\hat{\mathbf{B}}_{k_i}$, $\hat{\mathbf{C}}_{k_i}$, $\mathbf{D}_{k_i}$ and $\mathbf{S}_i$ are the decision variables, and the terms denoted $\bigstar$ are induced by symmetry. Finally, the controller matrices are computed with the following expressions:

$$\begin{aligned}\mathbf{A}_{k_i} = \mathbf{N}^{-1}(&\hat{\mathbf{A}}_{k_i} - \mathbf{X}(\mathbf{A}_i - \mathbf{B}_2\mathbf{D}_{k_i}\mathbf{C}_2)\mathbf{Y} - \hat{\mathbf{B}}_{k_i}\mathbf{C}_2\mathbf{Y}\\ &- \mathbf{X}\mathbf{B}_2\hat{\mathbf{C}}_{k_i})\mathbf{M}^{-\mathrm{T}},\end{aligned} \tag{7}$$

$$\mathbf{B}_{k_i} = \mathbf{N}^{-1}(\hat{\mathbf{B}}_{k_i} - \mathbf{X}\mathbf{B}_2\mathbf{D}_{k_i}), \tag{8}$$

$$\mathbf{C}_{k_i} = (\hat{\mathbf{C}}_{k_i} - \mathbf{D}_{k_i}\mathbf{C}_2\mathbf{Y})\mathbf{M}^{-\mathrm{T}}, \tag{9}$$