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Dynamic companion harmonic circuit models for analysis of power systems with embedded power electronics devices

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ABSTRACT

In this paper a methodology that extends the dynamic harmonic domain (DHD) analysis of large networks is presented. The method combines DHD analysis and discrete companion circuit modeling resulting in a powerful analytic technique called dynamic companion harmonic circuit modeling. It provides for a complete dynamic harmonic analysis of the system while preserving the advantages of discrete companion circuit models. The methodology is illustrated by its application to a three-node power system, where reactive power compensation is achieved using a fixed-capacitor, thyristor-controlled reactor (FC-TCR) and its control system.

1. Introduction

Modern electric networks include power electronics in the form of flexible AC transmission system (FACTS) devices, high voltage DC (HVDC) transmission links, and custom power system (CUPS) devices. As a result several challenges have arisen in the modeling and analysis of modern electric network. This has been exacerbated by the inclusion of alternative energy sources and storage devices connected to the electric grid by means of power electronics converters. Two of the challenges are the modeling for transient and harmonic analysis in large networks. For transient analysis of electric networks, software such as Electro-Magnetic Transients Program (EMTP), PSCAD/EMTDC, and Simplorer use circuit discrete models, resulting in a general nodal circuit formulation that allows time-domain analysis [1,2]. On the other hand, the method of dynamic average models [2] is a powerful computational tool for large and small signal analysis of power systems, but a limitation of this method is that the harmonics generated by the switching elements are not considered. The DHD method has been shown to be a powerful tool for the dynamic analysis of power elements [3–10]. In this paper a methodology that extends the dynamic harmonic domain (DHD) analysis of large networks is presented. The method combines DHD analysis and discrete companion circuit modeling resulting in a powerful analytic technique called dynamic companion harmonic circuit modeling. It provides for a complete dynamic harmonic analysis of the system, while preserving the advantages of discrete companion circuit models.

The methodology is illustrated by its application to a three-node power system, where the reactive power is compensated by a fixed-capacitor, thyristor-controlled reactor and its control system which controls the firing angle of the thyristors. Although the DHD method solves the differential equations of the system for the harmonics over time, the proposed method incorporates discrete companion circuit modeling [11,12] into the DHD method and transforms the differential equations into algebraic equations. The discrete models obtained in this manner are admittance matrices instead of purely resistive elements, which is the case when time-domain analysis is used. Also, a control system for the voltage control of the FC-TCR is included.

1.1. FC-TCR considerations

A common FACTS device is the static VAR compensator, and its most important property is its ability to maintain an approximately constant voltage at its terminals by continuous adjustments of the reactive power that it exchanges with the power system. Unfortunately, this compensation is based on circuit controllers that distort the waveform of the uncompensated voltages from their ideal sinusoidal form. Nevertheless, the static compensator is commonly used and studied assuming sinusoidal voltages.

Several papers have proposed solutions for the harmonic distortion caused by thyristor-controlled reactors (TCRs). A multiphase harmonic load flow has been described in Ref. [13], and a frequency

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domain TCR model is presented in Ref. [14]. In these papers, the control of voltage is achieved by an iterative scheme of adjusting the firing angle based on harmonic analysis that leads to steady state operation. Also, a steady state analysis based on the use of harmonic domain admittance matrices and functions is presented in Ref. [15]. In Ref. [5] the dynamic harmonic behavior of the system is presented, but the control action of the compensator is not considered.

2. Dynamic harmonic domain

The DHD methodology is based on orthogonal bases, complex Fourier series, matrix differential operators, and the approximation of operators. The main idea underlying the DHD is that a function $x(t)$ can be approximated by the time-dependent complex Fourier series [3–6]:

$$x(t) = \sum_{h=-\infty}^{h=\infty} X_h(t)\, e^{jh\omega_0 t} \tag{1}$$

The complex Fourier coefficients $X_h(t)$ are a function of time. Each coefficient is calculated using:

$$X_h(t) = \frac{1}{T}\int_{t}^{t+T} x(\tau)\, e^{-jh\omega_0\tau} d\tau \tag{2}$$

where $\omega_0 = 2\pi/T$, $\tau \in [t, t+T]$ and T is the period of time under consideration. Eq. (2) gives the time-evolution of the harmonics as a window of length T slides over the waveform $x(t)$. In Refs. [3–6] it is shown that the state-space equation $dx(t)/dt = a(t)x(t) + b(t)u(t)$ can be represented in the DHD by

$$\frac{d\mathbf{X}(t)}{dt} = [\mathbf{A}(t) - \mathbf{D}]\mathbf{X}(t) + \mathbf{B}(t)\mathbf{U}(t) \tag{3}$$

The vector $\mathbf{X}(t)$ is the state variable whose components are the harmonics coefficients of $x(t)$; the vector $\mathbf{U}(t)$ is the system input whose components are the harmonic coefficients of $u(t)$. The matrices $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are Toeplitz matrices whose elements are the harmonic coefficients of $a(t)$ and $b(t)$, respectively. The matrix $\mathbf{D}$ is a differential operator. The expression in (3) gives the dynamic evolution of the harmonic coefficients of the periodic variable $x(t)$. Also, the steady state response of (3) is obtained when the derivative of the state variable is equal to zero, that is, when $d\mathbf{X}(t)/dt = 0$. In steady state the explicit dependence on time in (3) can be suppressed yielding $0 = [\mathbf{A} - \mathbf{D}]\mathbf{X} + \mathbf{BU}$ or

$$\mathbf{X} = -[\mathbf{A} - \mathbf{D}]^{-1}\mathbf{BU} \tag{4}$$

Eqs. (3) and (4) are the basis of dynamic harmonic domain analysis. Eq. (3) gives the evolution in time of the harmonics of the system during the transient period and (4) establishes the condition for steady state. The steady state solution from (4) can be used as an initial condition when solving (3). This characteristic is very important since in many cases it is difficult to obtain a steady state solution of a dynamic system.

3. Companion harmonic circuit models

The solution of the differential equation representing an ideal inductor or capacitor can be approximated by applying the trapezoidal rule for integration to the equation. The difference equation which results can then be represented by a Norton equivalent circuit comprised of an admittance and a current source. The equivalent circuit is called the associated discrete circuit model or companion circuit model of the circuit element. Each of the inductive or capacitive elements in the electrical network of a power system can be converted to its companion circuit model before setting up the equations which describe the entire network [11,12]. The companion circuit model approach can be combined with the powerful methodology of dynamic harmonic domain analysis for transient and steady state calculations in the harmonic domain. It should be noted that a companion circuit model can be derived from any implicit numerical integration formula associated with a circuit element or device [11,12]. In this section, we shall derive the companion harmonic circuit models for the inductors, capacitors, and a thyristor-controlled reactor using their defining differential equations in the DHD.

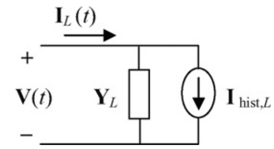

Fig. 1. Companion harmonic circuit model of an ideal inductor.

3.1. Companion harmonic circuit modeling

The differential equation that represents the terminal characteristics of an ideal inductor L is given by

$$v(t) = L\frac{di_L(t)}{dt} \tag{5}$$

According to Eq. (3), Eq. (5) may be expressed in the DHD by

$$\frac{d\mathbf{I}_L(t)}{dt} = -\mathbf{D}\mathbf{I}_L(t) + \frac{1}{L}\mathbf{V}(t) \tag{6}$$

Applying the trapezoidal numerical integration approximation to Eq. (6), the following expression is obtained:

$$\frac{\mathbf{I}_L(t) - \mathbf{I}_L(t-\Delta t)}{\Delta t} = -\mathbf{D}\frac{\mathbf{I}_L(t) + \mathbf{I}_L(t-\Delta t)}{2} + \frac{1}{L}\frac{\mathbf{V}(t) + \mathbf{V}(t-\Delta t)}{2} \tag{7}$$

where Δt is the integration time step. Solving for $\mathbf{I}_L(t)$ gives the companion harmonic circuit model in the Norton equivalent form:

$$\mathbf{I}_L(t) = \mathbf{Y}_L\mathbf{V}(t) + \mathbf{I}_{\text{hist},L} \tag{8}$$

where $\mathbf{Y}_L$ is a complex admittance matrix given by

$$\mathbf{Y}_L = \frac{\Delta t}{L}[2\mathbf{U}_I + \Delta t\mathbf{D}]^{-1} \tag{9}$$

$\mathbf{U}_I$ is the identity matrix, and the history term $\mathbf{I}_{\text{hist},L}$ is given by

$$\mathbf{I}_{\text{hist},L} = \frac{L}{\Delta t}\mathbf{Y}_L\,[2\mathbf{U}_I - \Delta t\mathbf{D}]\,\mathbf{I}_L(t-\Delta t) + \mathbf{Y}_L\mathbf{V}(t-\Delta t) \tag{10}$$

Eq. (8) is the companion harmonic circuit model of the inductor shown in Fig. 1. This model is required for the transient analysis of the system.

For a capacitance C, the governing differential equation is

$$i_C(t) = C\frac{dv(t)}{dt} \tag{11}$$

In the DHD this equation is represented by

$$\frac{d\mathbf{V}(t)}{dt} = -\mathbf{D}\mathbf{V}(t) + \frac{1}{C}\mathbf{I}_C(t) \tag{12}$$

The Norton equivalent circuit for a capacitor is determined in a manner similar to that used for an inductor and is given by

$$\mathbf{I}_C(t) = \mathbf{Y}_C\mathbf{V}(t) + \mathbf{I}_{\text{hist},C} \tag{13}$$

where

$$\mathbf{Y}_C = C\left[\frac{2\mathbf{U}_I}{\Delta t} + \mathbf{D}\right] \tag{14}$$