ARTICLE IN PRESS

Flow Measurement and Instrumentation ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Flow Measurement and Instrumentation

journal homepage: www.elsevier.com/locate/flowmeasinst

A mesh-less method to solve electrical resistance tomography forward problem using singular boundary distributed source method

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ARTICLE INFO

Article history:
Received 2 December 2014
Received in revised form
22 May 2015
Accepted 23 June 2015

Keywords:
Meshless methods
Improved boundary distributed source method
Singular boundary method
Electrical resistance tomography
Forward problem

ABSTRACT

This paper presents a novel mesh-less method called singular boundary distributed source (SBDS) method to solve the electrical resistance tomography (ERT) forward problem. Mesh-less methods are mathematically simple and computationally efficient i.e. same accuracy that of mesh or domain based methods can be achieved with less computation time. The domain boundary is discretized such that the source and field points coincide on the same boundary and the solution is expressed as a linear combination of fundamental solution of governing equation. The singularity appearing due to coinciding of source and field points is addressed using a hybrid approach. The coefficients corresponding to Neumann boundary are evaluated using the improved distributed source method (IBDS) while the coefficients corresponding to Dirichlet boundary conditions are evaluated using inverse interpolation technique. The hybrid SBDS with complete electrode model is formulated for ERT forward problem. The proposed SBDS method has better accuracy and convergence as compared to other mesh-less methods. Especially, the SBDS method solution is not dependent on distributed source radius therefore is stable compared to the IBDS method. The proposed SBDS method is tested with numerical experiments and the performance is compared against boundary discretization methods such as the IBDS and boundary element methods.

1. Introduction

Electrical resistance tomography (ERT) is a non-intrusive imaging modality that reconstructs the conductivity/resistivity distribution inside the object by placing the electrodes on its surface [1]. Small amount of electric current is injected through the attached electrodes and the excited voltages due to presence of substances in the domain of interest are measured. Based on the current–voltage relationship the internal distribution is reconstructed [2]. The cross-sectional images from ERT provide valuable information that help in process monitoring. With respect to flow measurement application using ERT, it can be used to estimate the velocity and flow rates of fluids [3,4]. Also, it is used to control the mixing process by measuring the mixing index. Another potential application of ERT is that the boundary of anomalies are estimated assuming the prior knowledge of conductivity of fluids flowing through process pipe [5].

The ERT image reconstruction problem involves iterating the forward and inverse solution. Forward problem of ERT involves solving the governing equation derived from the Maxwell equations subjected to boundary conditions to compute the potential distribution and boundary voltages. Inverse problem is about estimating the internal distribution by minimizing the difference between the measured and calculated voltages. The forward problem is generally solved using numerical methods except for simple cases such as homogeneous and concentric. Numerical methods such as the finite element method (FEM), finite difference method (FDM), boundary element method (BEM) are often used as forward solvers to compute the boundary voltages. The finite element method discretizes the domain into finite elements and the solution is approximated as a linear combination of known functions [6,7]. In the boundary element method the boundary is discretized and the solution is transformed from the domain to

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http://dx.doi.org/10.1016/j.flowmeasinst.2015.06.027
0955-5986/

Please cite this article as: A. Kumar Khambampati, et al., A mesh-less method to solve electrical resistance tomography forward problem using singular boundary distributed source method, Flow Measurement and Instrumentation (2015), http://dx.doi.org/10.1016/j.flowmeasinst.2015.06.027

boundary elements [8,9]. In solving the ERT problem, the cost functional is defined in terms of voltage difference and the parameter is estimated by minimizing the cost function in the least square sense [2,10]. Thus, it involves computing the forward solution at each iteration step. Therefore, it is necessary to have an accurate and fast forward solution for real-time monitoring of the process.

Mesh-less methods that are mathematically simple and computationally efficient have attracted the scientific community and are applied to various engineering applications [11–13]. Mesh-less methods can attain same accuracy as domain and boundary discretization methods with less computational time [11]. Mesh-less methods evaluates the solution based on nodes therefore are really mesh-less and does not involve any meshing thus it reduces the computational time. Moreover, they do not need complex integration and can be easily extended to higher dimensions. Among the mesh-less methods, the method of fundamental solution (MFS) has attracted the scientific community due to its simplicity in use. In MFS, the solution is expressed as a linear combination of fundamental solution across the discretized nodes on the boundary [14,15]. To avoid singularity while evaluating the fundamental solution, MFS uses a fictitious circle to place the source points. The solution is highly dependent on the choice of this fictitious circle. There is no mathematical framework as of now that determines the radius of fictitious circle and especially it is difficult with complex shaped domain.

Recently developed mesh-less methods such as the modified method of fundamental solution (MMFS) [16], singular boundary method (SBM) [17] and boundary distributed source method (BDS) [18] places the source and collocation points on the same physical boundary thus the need of fictitious circle is avoided. However, to treat the singularity issue when the source and field point coincide each method adopts different strategy. In MMFS method an origin intensity factor is introduced and numerical integration is done to evaluate the origin intensity factor. In BDS a distributed source is considered and integration is done over the surface of the distributed surface. Similar to MMFS, in SBM, the origin intensity factor is introduced but to evaluate the intensity factor, inverse interpolation method with subtracting and adding back method is used. While computation of the system matrix, the diagonal elements of Neumann boundary conditions in BDS method and diagonal elements of Dirichlet boundary conditions in SBM method are solved using an indirect method therefore they are computationally intensive [18,19]. The improved boundary distributed source (IBDS) [20, 21] method considers analytic evaluation of both Dirichlet and Neumann type boundary conditions therefore it is accurate and stable compared to BDS method. The IBDS solution is affected by the radius of the distributed source especially in case of multiple connected domains. There is no method as of now to determine the optimal distributed source radius to have reliable solution.

In this paper, a hybrid method called the singular boundary distributed source method (SBDS) is proposed to solve the ERT forward problem. SBDS with complete electrode model is formulated for ERT forward problem. The system matrix corresponding to Neumann boundary conditions are solved using the IBDS approach while the diagonal elements of Dirichlet boundary conditions are solved using an inverse interpolation technique. SBDS has both the features of IBDS and SBM methods hence has better accuracy and convergence. Moreover, it is noticed that the solution is not dependant on radius of the distributed source. The proposed SBDS method is tested with numerical experiments and the performance is compared against IBDS method and BEM.

2. ERT forward problem and mesh-less methods

2.1. Complete electrode model

In ERT, the object circumference contains L electrodes e_l (l=1,2,...,L) that are discretely attached to its surface $\partial\Omega$. Currents of magnitude I_l (l=1,2,...,L) are applied through these electrodes and the resulting voltages on the electrodes are measured. Let us consider the case where the computational domain Ω contains an anomaly with conductivity σ_a occupying region D embedded inside the homogeneous background with a fluid having conductivity σ_b. The relation between conductivity distribution σ and voltage potential u is governed by a Laplace equation which is derived from Maxwell's equation [2]

$$\nabla\cdot\left[\sigma_b + (\sigma_a - \sigma_b)\chi_D(p)\right]\nabla u(p) = 0 \;\text{ for } p \in \Omega \text{ and } D \subset \Omega \tag{1}$$

where p is the spatial location (x, y) in the computational domain Ω and χ_D is the characteristic function that has value 1 inside the region D and zero otherwise. The boundary conditions based on the complete electrode model (CEM) that considers contact impedance and shutting effect are as follows [22]:

$$\frac{\partial u(p)}{\partial \nu} = 0 \text{ for } p \in \partial\Omega_G = \partial\Omega \backslash \cup_{l=1}^{L} e_l \tag{2}$$

$$\int_{e_l} \sigma_b \frac{\partial u(p)}{\partial \nu} dS = I_l \;\text{ for }\; p \in e_l,\; l = 1, 2, \ldots, L \tag{3}$$

$$u(p) + z_l\sigma_b \frac{\partial u(p)}{\partial \nu} = U_l \;\text{ for } p \in e_l,\; l = 1, 2, \ldots, L \tag{4}$$

where z_l is the effective contact impedance between the l–th electrode and electrolyte, U_l is the boundary voltage measured on the l–th electrode and ν is outward unit normal. The regions for outer boundary, electrode and gap are represented as $\partial\Omega = \partial\Omega_G \cup \partial\Omega_E$, $\partial\Omega_E = \cup_{l=1}^{L} e_l$, and $\partial\Omega_G = \cup_{l=1}^{L} g_l$. There also exists an interfacial condition at the boundary of anomaly ∂D, i.e.

$$u(p)|_{\partial D^-} = u(p)|_{\partial D^+} \text{ and } \sigma_a \frac{\partial u(p)}{\partial \nu}\bigg|_{\partial D^-} = \sigma_b \frac{\partial u(p)}{\partial \nu}\bigg|_{\partial D^+} \tag{5}$$

2.2. Singular boundary method

In singular boundary method (SBM), the solution for potential distribution and its derivative is expressed as a linear combination of fundamental solution of the governing equation given by [16]

$$u(p_i) = \sum_{j=1}^{N} \alpha_j G(p_i, p_j) \tag{6}$$

$$q(p_i) = \frac{\partial u(p_i)}{\partial \nu(p_i)} = \sum_{j=1}^{N} \alpha_j \frac{\partial G(p_i, p_j)}{\partial \nu(p_i)} \tag{7}$$

where p_i is the ith collocation point, p_j is the jth source point, α_j the jth unknown intensity of the distributed source at p_j, N the numbers of source points and G is the fundamental solution of the Laplace equation defined as,

$$G(p_i, p_j) = -\frac{1}{2\pi} \ln\left|p_i - p_j\right| \tag{8}$$

The SBM interpolation formula is given by [13]

$$u(p_i) = \sum_{j=1, i\neq j}^{N} \alpha_j G(p_i, p_j) + \alpha_i u_{ii} \tag{9}$$

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