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ROBUSTNESS ISSUES IN CONTINUOUS-TIME SYSTEM IDENTIFICATION FROM SAMPLED DATA

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Abstract: This paper explores the robustness issues that arise in the identi cation of continuous-time systems from sampled data. A key observation is that, in practice, one cannot rely upon the delity of the model at high frequencies. This implies that any result which implicitly or explicitly depends upon the folding of high frequency components down to lower frequencies will be inherently non-robust. We illustrate this point by referring to the identi cation of continuous-time auto-regressive stochastic models from sampled data. We argue that traditional approaches to this problem are sensitive to high frequency modelling errors. We also propose an alternative maximum likelihood procedure in the frequency domain, which is robust to high frequency modelling errors.

Keywords: continuous-time systems, parameter estimation, stochastic systems, robust estimation, sampled data

1. INTRODUCTION

Identi cation of continuous-time systems is a problem of considerable importance in various disciplines such as economics, control, fault detection and signal processing. In recent years, there has been an increased interest in the problem of identifying continuous-time models (Rao and Garnier, 2002; Garnier *et al.*, 2003; Ljung, 2003). Even though it is theoretically possible to carry out system identi cation using continuous-time data (Young, 1981; Unbehauen and Rao, 1990), this will generally involve analogue operators to emulate time derivatives and will thus usually be impractical. Thus, one is usually forced to work with sampled data (Sinha and Rao, 1991; Pintelon and Schoukens, 2001). In this context, one might hope that if one samples quickly enough then the di erence between discrete and continuous processing would be vanishing small. There are indeed many cases which support this hypothesis — see, for example, (Middleton and Goodwin, 1990; Feuer and Goodwin, 1996; Goodwin *et al.*, 2001).

The above discussion can, however, lead to a false sense of security when using sampled data. A well known instance where *naive* use of sampled data can lead to erroneous results is in the identi cation of continuous-time stochastic systems where

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the noise model has relative degree greater than zero. In the latter case, it has been shown in (Wahlberg, 1988) that the sampled data model will have *sampling zeros*. These are the stochastic equivalent of the well-known sampling zeros that occur in deterministic systems of relative degree greater than one (Åström *et al.*, 1984). The stochastic sampling zeros play a crucial role in obtaining unbiased parameter estimates in the identification of such systems from sampled data. The reason is that most identification procedures rely upon *whitening* of the noise, an operation which is sensitive to the sampling zeros of continuous-time systems of non zero relative degree.

A particular case of the above problem has been studied in detail in (Söderström *et al.*, 1997; Larsson and Söderström, 2002; Larsson, 2003). In particular, these papers deal with continuous-time auto-regressive (CAR) system identification from sampled data. Such systems have relative degree n, where n is the order of the auto-regressive process. It has been shown that if one ignores the stochastic sampling zeros, *e.g.*, by using ordinary least squares, a clear bias will appear in the parameter estimates, even when using fast sampling rates (Söderström *et al.*, 1997).

In the current paper we further explore the circle of ideas outlined above. We pay particular attention to the impact of high frequency modelling errors on continuous-time system identification when using sampled data. We show that high frequency modelling errors can be equally as catastrophic as ignoring sampling zeros. Thus we argue that one should always define a *bandwidth of fidelity* of a model and ensure that the model errors outside that bandwidth do not have a major impact on the identification results. This leads us to develop a frequency domain identification procedure which we show is insensitive to both relative degree and unmodelled high frequency poles.

2. BACKGROUND TO THE IDENTIFICATION OF CAR SYSTEMS

The ideas presented in this paper are equally applicable to all continuous-time identification problems. However, to be specific we will focus primarily on the case of CAR system identification from sampled data.

In (Larsson and Söderström, 2002), estimation of the parameters of a CAR system is performed by using a filtered least squares procedure. In fact, the prefilter applied to the data is closely related to the asymptotic sampling zeros described in (Wahlberg, 1988) (for stochastic models, and in (Åström *et al.*, 1984), for the deterministic case). This is an elegant and insightful solution to the problem. However, the asymptotic location of the sampling zeros depend on the relative degree of the continuous-time plant description. At this point our claim about a *bandwidth of validity* for this model becomes relevant since relative degree may be an ill-defined quantity for continuous-time systems.

This kind of issues has been previously illustrated, for example, by the same authors in the context of deterministic control (Yuz *et al.*, 2004). Here we extend these ideas to the identification problem.

We consider a CAR system described by:

$$A_c(\rho)y(t) = \dot{v}(t) \tag{1}$$

where $A_c(\rho)$ is a polynomial in the differential operator $\rho = \frac{d}{dt}$, *i.e.*

$$A_c(\rho) = \rho^n + a_{n-1}\rho^{n-1} + \ldots + a_0 \tag{2}$$

In equation (1) the term $\dot{v}(t)$ represents a continuous-time white noise process.

Remark 1. We already notice the source of some difficulties since the process $\dot{v}(t)$ does not exist in any meaningful sense. Indeed, equation (1) should actually be written as a stochastic differential equation driven by a process with independent increments, that is, *Brownian motion* or *Wiener* process (Øksendal, 2003). Indeed, a continuous-time *white noise* (CTWN) process is a mathematical abstraction and does not physically exist (Jazwinski, 1970), but it can be approximated to any desired degree of accuracy by conventional stochastic processes with broad band spectra (Kloeden and Platen, 1992). Note, however, that the difference between a *broad band* spectra and *white noise* is equivalent to a particular form of high frequency modelling error. This is the key issue of relevance in the current paper.

If we treat equation (1) appropriately then it is possible to derive an exact discrete-time system that describes the samples of $y(t)$ (Wahlberg, 1988). This model takes the following generic form:

$$A_d(q^{-1})y(k\Delta) = B_d(q^{-1})w_k \tag{3}$$

where w_k is a discrete-time white noise process, and A_d and B_d are polynomials in the backward shift operator q^{-1}.

It is readily shown that the polynomial $A_d(q^{-1})$ in equation (3) is *well behaved* in the sense that it converges naturally to its continuous-time counterpart. This relationship is most readily portrayed if the model is rewritten in the equivalent delta form (Middleton and Goodwin, 1990):

$$A_\delta(\gamma) = \gamma^n + a_{n-1}\gamma^{n-1} + \ldots + a_0 \tag{4}$$