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FRICTION IDENTIFICATION BASED UPON THE LUGRE AND MAXWELL SLIP MODELS ⋆

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Abstract: Three friction identification methods, designated as the LuGre (LG) method, the Non-Linear Regression (NLR) method, and the Dynamic Non-Linear Regression with direct application of the eXcitation (DNLRX) method, are postulated. The first employs the LuGre model structure, the second the basic Maxwell Slip model structure, and the third an extended version of it. The Maxwell Slip model structure accounts for the presliding hysteresis with nonlocal memory, but is confined to providing constant sliding friction. This limitation is circumvented by the extended version postulated, where additional dynamics are introduced. In all methods identification is based upon signals obtained from a single experiment, thus circumventing the need for multiple experiments. The methods are assessed via laboratory signals, and the DNLRX is shown to achieve the best overall performance, followed by the NLR and, finally, the LG method. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Friction is a major nonlinear phenomenon that may lead to tracking errors, limit cycles, stick and slip motion, and so on. Its behavior may be distinguished into two operating regimes: The *presliding (micro-slip)* and the *sliding* regimes. In the first the adhesive forces are dominant, and friction depends, among other factors, on the past extreme values of the displacement, thus exhibiting hysteresis within nonlocal memory (Swevers *et al*., 2000). This hysteresis disappears upon switching from the *presliding* to the *sliding* regime. Within the latter regime friction depends mainly on the velocity, and various nonlinear phenomena (such as the *Stribeck effect*, *frictional lag* and so on) are exhibited (Armstrong-Hélouvry *et al*., 1994).

Accurate friction modelling based upon the first principles and material / surface properties is not possible to date. Thus, identification methods based upon experimentally obtained signals are typically used. Classical methods relate friction directly to velocity and / or displacement, and attempt identification via either time domain (Armstrong-Hélouvry *et al*., 1994; Kim *et al*., 1996) or frequency domain techniques (Chen *et al*., 2002). The obtained models generally tend to oversimplify the actual frictional behavior.

More elaborate methods relate friction to velocity and/or displacement via internal (unobservable) *state variables*. The underlying dynamics is better described, but the identification becomes more challenging. In general, identification is achieved by separating the unknown parameters into *static* and *dynamic*, corresponding to the sliding and presliding regimes, respectively, and performing dedicated experiments in each regime. A notable class of such methods relies on the LuGre model (Canudas de Wit and Lischin-

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sky, 1997; Hensen *et al.*, 2002) and its extension, referred to as the Elastoplastic friction model (Dupont *et al.*, 2002). An alternative method is based upon the Leuven friction model (Swevers *et al.*, 2000), which is similar to the LuGre model, but extended for capturing the presliding hysteresis with nonlocal memory.

The current study *aims* at identifying the combined presliding / sliding friction dynamics based upon the LuGre and Maxwell Slip model structures. Three identification methods, designated as the LG method (LuGre), the NLR (NonLinear Regression) method, and the DNLRX (Dynamic NonLinear Regression with direct application of the eXitation) method, are formulated and assessed. The first employs the LuGre model structure. The second employs the basic Maxwell Slip model structure, and is thus capable of accounting for the presliding hysteresis with nonlocal memory (Lampaert *et al.*, 2002), but may only provide constant sliding friction. The third circumvents this limitation by employing a presently formulated extended form of the Maxwell Slip model structure that makes use of two finite impulse response filters.

In all methods identification is based upon a single pair of displacement – friction signals. The experimental procedure is thus simplified, as the usual need for several dedicated experiments is circumvented.

2. MODEL STRUCTURES

2.1 *The LuGre Model Structure*

The LuGre model structure (Canudas de Wit *et al.*, 1995) contains an unobservable state variable z, representing the average deflection of the elastic "bristles" that are responsible for friction generation. It accounts for most of the observed frictional dynamics, but the presliding hysteresis with nonlocal memory is not represented (Swevers *et al.*, 2000).

The LuGre model features the nonlinear state equation:

$$\frac{dz}{dt} = v - \frac{|v|}{s(v)} \cdot z \tag{1}$$

and an output equation for approximating the frictional force as follows:

$$F_{LG} = \sigma_0 \cdot z + \sigma_1 \cdot \frac{dz}{dt} + \sigma_2 \cdot v \tag{2}$$

with v designating velocity, σ_0 an equivalent stiffness, and σ_1, σ_2 the micro-viscous and viscous friction coefficients, respectively. $s(v)$ designates a user-defined function that models the constant-velocity behavior. The following parametrization, similar to a typical one (Armstrong-Hélouvry *et al.*, 1994), is presently adopted for $s(v)$:

$$s(v) = a_1 + \frac{a_2}{1 + \left(\frac{|v|}{v_s}\right)^{\mu}}, \; a_1 = \frac{F_c}{\sigma_0}, \; a_2 = \frac{F_s - F_c}{\sigma_0} \tag{3}$$

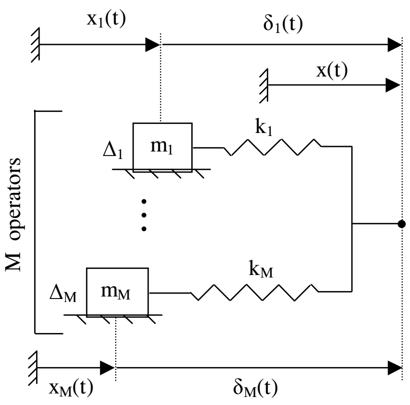

Fig. 1. The basic Maxwell Slip model structure.

with F_c and F_s designating the *Coulomb* and *static* friction, respectively, v_s the *Stribeck* velocity, and μ a parameter providing for extra modelling flexibility.

2.2 *Structures Based Upon the Maxwell Slip Model*

The Basic Structure. The basic Maxwell Slip model structure consists of M elasto-slide operators in parallel configuration, which are subject to a common displacement excitation $x(t)$ [Fig. 1]. Each operator has negligible inertia, its own linear stiffness k_i, and maximum spring deformation Δ_i (threshold). For spring deformation smaller, in magnitude, than Δ_i ($|\delta_i(t)| < \Delta_i$) the operator *sticks*; otherwise it *slips* ($|\delta_i(t)| = \Delta_i$). The whole system sticks (presliding regime) iff at least one operator sticks ($\exists j \in [1, M]$: $|\delta_j(t)| < \Delta_j$), and slides (sliding regime) iff all operators slip ($|\delta_i(t)| = \Delta_i, \forall i$).

In mathematical terms, the model is described by a set of nonlinear state equations (Rizos and Fassois, 2004):

$$\delta_i(t+1) = \text{sgn}[x(t+1) - x(t) + \delta_i(t)] \cdot \cdot \min\{|x(t+1) - x(t) + \delta_i(t)|, \Delta_i\} \tag{4}$$

with $i = 1, \ldots, M$, while the friction is approximated as the sum of the operators' forces:

$$F_M(t) = \sum_{i=1}^{M} k_i \cdot \delta_i(t) \tag{5}$$

with $t = 1, 2, \ldots$ referring to (normalized) discrete time.

Among the main advantages of this basic structure is simplicity, physical interpretation, and its capability of describing the presliding hysteresis with nonlocal memory (Lampaert *et al.*, 2002). Yet, the model accounts for constant sliding (Coulomb) friction only [see Eq. (5) and recall that the system slides iff $|\delta_i(t)| = \Delta_i, \forall i$, that is iff *all* operators slip]. It is evident that this constraint may impair modelling accuracy, hence a proper extension is introduced in the sequel.