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Numerical study of the heat transfer and electro-thermo-convective flow patterns in dielectric liquid layer subjected to unipolar injection

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ABSTRACT

In this article we study the electro-thermal convection in a dielectric liquid layer placed between two electrodes and subjected to the simultaneous action of an electric field and a thermal gradient. The full set of equations describing the electro-thermo-convective phenomena is directly solved using a finite volume method. We first heat the liquid from below at time $t = 0$, wait for the thermal steady state and then inject the electric charges by applying the electric potential. The development of the electro-convective motion is analysed in detail in two cases: 1) strong injection from the lower electrode, 2) strong injection from the upper one. We also study the heat transfer enhancement due to electro-convection. The evolution in time of the Nusselt number *Nu* for different combinations of the two usual non-dimensional parameters associated to the electro-thermo-convection phenomena (Rayleigh number *Ra* and the electrical parameter *T*) is also given and analysed.

1. Introduction

The combined effects of an electric field and a thermal gradient simultaneously applied to a horizontal dielectric liquid layer leads to very complex physical interactions in the flow. It has been shown experimentally [1,2] that the heat transfer across an insulating liquid can be increased by one order of magnitude. This increase is due to the development of secondary motions that result from two principal body forces: the Coulomb force acting on any free electric charge present in the liquid and the dielectric force. Here we consider a dielectric liquid of very low conductivity so that we can consider that the free space charge only come from ion injection mechanisms at the electrodes (by electrochemical reactions). The amount of electric charge in the liquid due to injection mechanisms at the interfaces is a lot greater (more than one order of magnitude) than the one induced in the volume by electrical conduction mechanisms.

In Electro-Hydro-Dynamic (EHD) the convection induced by charge injection in a dielectric liquid is a problem as fundamental as the one of Rayleigh-Benard in non isothermal fluid mechanics. The action of the electric field on the space charge density arising from a unipolar injection (electric charges coming from only one electrode, which is the case in this article) has the same destabilizing role than the thermal field when the fluid is heated from below in Rayleigh-Benard problems [3]. However both convections are not identical from a physical point of view. The mechanisms at the origin of the motion of the fluid are quite different in both cases: in Rayleigh-Benard convection the heat transfer is governed by thermal diffusion whereas the ion migration is the relevant mechanism in the electric charge transfer in electro-convection. When the space charge only results from ion injection, the coupling between the conservation equations of momentum, electric charge and energy is ensured via the Coulomb and the buoyancy forces. This coupling results from the direct interaction between velocity, temperature and charge perturbations and from the non direct interaction between the velocity and the charge. Most of the authors who have been working theoretically so far on electro or electro-thermo-convective problems in horizontal planar layers of dielectric liquids chose a stability analysis approach [4–7]. Only a few numerical simulations have been attempted on pure EHD convection problems [8–10] over the last decade.

In [11], for the first time, we solve the Electro-Hydro-Dynamic problem coupled with the energy equation in a 2D cavity and developed a direct numerical simulation based on a finite-volume method. Here we focus on the convective mechanisms responsible for fluid motion when the charges are injected from the lower

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or upper electrode into an insulating liquid. The spatio-temporal evolution of the electro-thermo-convective flow in the dielectric liquid layer is analysed in detail. The influence of the induced electro-convection on the heat transfer is studied by the mean of the time evolution of the Nusselt number.

2. Statement of the problem

2.1. Basic governing equations

We consider a dielectric liquid layer of thickness H enclosed between two electrodes of length L_x. The layer is heated from below and subjected to a thermal gradient $\Delta\theta = \theta_0 - \theta_1$. The emitter electrode which injects a charge density q_0 can be either the lower electrode (Electrode 0) or the upper one (Electrode 1). Between the two electrodes a potential difference $\Delta V = V_0 - V_1$ is applied. Fig. 1 displays the case where the emitter electrode is Electrode 0.

The general set of equations expressing the conservation of: 1) the mass and momentum (Navier–Stokes equations) for an incompressible fluid with electrical and buoyancy effects; 2) the energy under the Boussinesq assumption; 3) the charge density and the divergence properties of the electric field (Maxwell–Faraday and Maxwell–Gauss equations) takes the following dimensionless form:

$$\nabla\cdot\vec{u} = 0 \tag{1}$$

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla\tilde{p} + \Delta\vec{u} + RTCq\vec{E} + \frac{Ra}{Pr}\theta\vec{e}_z \tag{2}$$

$$\frac{\partial\theta}{\partial t} + \vec{u}\cdot\nabla\theta = \frac{1}{Pr}\Delta\theta \tag{3}$$

$$\frac{\partial q}{\partial t} + \nabla\cdot(q(\vec{u} + R\vec{E})) = 0 \tag{4}$$

$$\Delta V = -Cq \tag{5}$$

$$\vec{E} = -\nabla V \tag{6}$$

where $\vec{u}$ is the velocity, $\tilde{p}$ the modified pressure which includes the pressure and the scalar from which the electrostriction force derives, q the volumic electric charge density in the liquid, $\vec{E}$ the electric field, ε the permittivity of the fluid, θ the absolute temperature, V the electric potential and $q\vec{E}$ the Coulomb force. We introduce the following non-dimensional scales: H for the length, the applied voltage $\Delta V = V_0 - V_1$ for the electric potential, $\Delta V/H$ for the electric field, q_0 for the charge density, $\varepsilon_0 K \Delta V^2/H^3$ for the current density, υ/H for the velocity (υ is the kinematic viscosity of the liquid), $\rho_0\upsilon^2/H^2$ for the pressure, $\theta/(\theta_0 - \theta_1$ for the temperature and

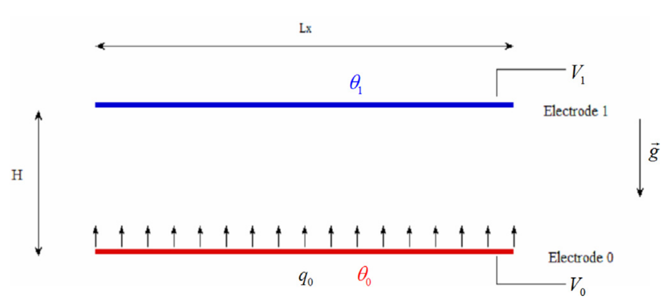

Fig. 1. Sketch of the physical domain.

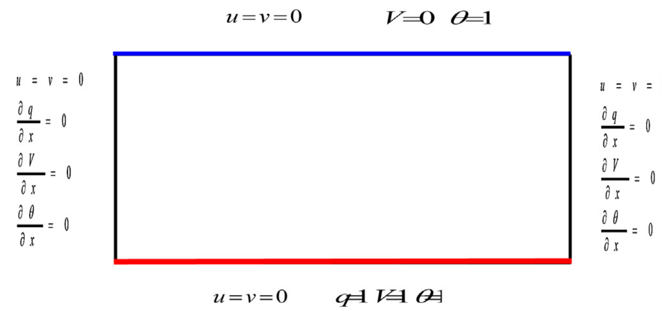

Fig. 2. Computational domain and boundary conditions.

H^2/υ for the time. The following dimensionless quantities defined as: $Ra = g\beta\Delta\theta H^3/\nu\kappa$, $T = \varepsilon\Delta V/\rho\nu K$, $C = q_0 H^2/\varepsilon\Delta V$, $M = \sqrt{\varepsilon/\rho K^2}$, $Pr = \nu/\kappa$ and $R = T/M^2$, are respectively the Rayleigh number with g the gravity and κ the thermal diffusivity of the liquid, T the electric instability parameter, C a measure of the injection level with q_0 the injected charge at Electrode 0, M the mobility parameter, Pr the Prandtl number and R the electric Reynolds number. In this study, we consider the case of a strong injection so that the dielectric force $-(1/2)\|\vec{E}\|^2\nabla\varepsilon$ is much lower than the Coulomb force and can be neglected [10,12].

2.2. Initial and boundary conditions

We start from rest with all the quantities set to zero. The whole simulation duration is $t = 50$, in dimensionless time. In order to well characterize the enhancement of heat transfer we first start to heat the liquid from below from time $t = 0$. When the thermal steady state is obtained, we apply the electric potential difference between the two electrodes so that the electric charges are injected in the bulk.

The horizontal walls are assumed impermeable as well as thermally and electrically perfectly conducting. We consider no-slip boundary conditions for the velocity on lateral and horizontal walls. The boundary conditions for the potential and the temperature in the case of heating and injecting from the lower electrode are depicted in Fig. 2.

3. Numerical method

The above set of coupled partial differential Eqs. (1)–(6) are discretized using a finite-volume approach. Full details on the

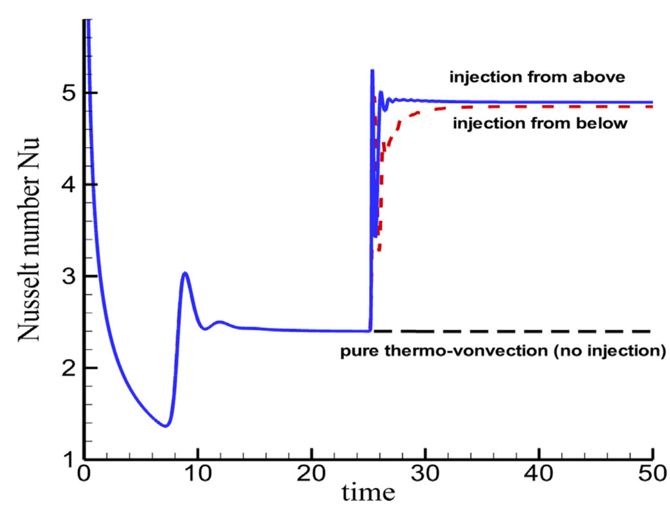

Fig. 3. Time evolution of Nusselt number, in three cases for $C = 10$, $T = 200$, $Ra = 10{,}000$, $Pr = 40$ and $M = 10$.