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Voltage calculations for annular tanks partly-filled with charged liquid

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ABSTRACT

A Bessel function expression is developed for the voltages produced when annular tanks (vertical axis cylindrical tanks with central conductors) are filled with liquids of uniform charge density. The expression is used to calculate the maximum surface voltage and this is compared with the maximum voltage predicted for tanks without a central conductor. Previous estimates of the percentage voltage reduction produced by a central conductor during tank filling have indicated a reduction of about 42% for practical tank dimensions. The new results, which are obtained with a more realistic model geometry, suggest a reduction of only about 29%.

1. Introduction

When a conductive-walled tank with an earthed central conductor (e.g. a truck compartment with a centrally-located dip tube or top-loading fill pipe) is filled with an electrostatically charged liquid, the presence of the central conductor reduces the maximum potential on the liquid surface. Voltage reductions have been observed experimentally in medium-sized vessels (defined as 1.3 m $\leq$ effective diameter $<$ 10 m [1]) but a range of voltage reduction factors (ratio of maximum potentials with and without central conductor) has been reported (see Section 5).

Historically, maximum tank filling rates for controlling electrostatic ignition hazards were determined for top loading in the presence of a central fill pipe and later an allowance for increased voltage was made when recommending filling rates in the absence of a central conductor [2]. Because of the experimental uncertainties in the observed degree of voltage reduction produced by a central conductor it is of interest to model the potentials attained with a central conductor to better understand the relationship expected between potentials with and without a central conductor and the parameters that could influence it.

This paper describes a model for the potentials attained in a partly-filled vertical axis cylindrical tank of finite height with conductive side wall, roof and base that has a central conductor consisting of a coaxial inner cylinder. The geometry, which may be termed an "annular tank" configuration, is shown in Fig. 1.

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2. Previous calculations

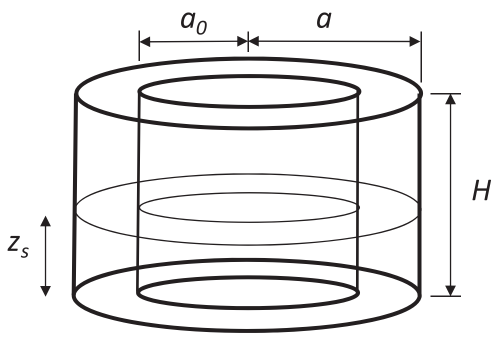
Liquid surface potentials in partly-filled rectangular [3] and vertical-axis cylindrical [4] tanks without central conductors have been calculated previously with some success using models based on uniform liquid charge density. The potentials predicted by these models during tank filling are of approximately the right magnitude [5] although the calculated maximum potential typically occurs earlier in the fill than the measured maximum.

Equivalent calculations have not been reported for tanks with central conductors. Simple theoretical estimates have been made of the expected scale of reduction [6] but they are based on rather crude models which calculate voltages in tanks that are completely full of liquid and either infinitely tall (simple analytical solutions covering all central conductor diameters) or of finite height (numerical solutions for limited number of geometries with potentials being considered at half height). The present work describes calculations equivalent to the Asano cylindrical tank model [4] but for annular tanks. The voltage reduction produced by the central conductor is calculated by comparing the annular tank results with results obtained from the original Asano model for cylindrical tanks with the same overall dimensions.

3. Calculation details

3.1. Outline of model

We have adapted Asano's model [4] for potentials in vertical-axis cylindrical tanks to cover annular tanks (Fig. 1). In the body of the paper we concentrate on the calculation of potentials at the liquid surface because these are the highest potentials that appear



NOTE: although a_0, z_s and H are shown here as tank dimensions they are used in the text for the dimensionless ratios formed by dividing the dimensions by the tank radius, a

Fig. 1. Annular tank geometry.

in the vapour space and hence are the most important for assessing electrostatic hazards. Expressions for the potentials at other locations in both the liquid and vapour spaces are, however, given in Appendix A.

Like Asano, we used separation of variables to solve Poisson's equation in the liquid space and Laplace's equation in the vapour space and hence expressed potentials in terms of the sum of products of radial Bessel functions and vertical hyperbolic functions. Our solutions have a similar form to Asano's (see Appendix A) but the radial functions involve Bessel functions of the second kind as well as of the first kind and the eigenvalues are no longer the zeros of the Bessel function, J_0. The vertical functions differ only in the modified eigenvalues.

The basic calculations give annular-tank liquid surface voltages in terms of tank dimensions, charge density and liquid depth. These have been supplemented by tank filling charging/relaxation calculations of the type done by Britton and Smith [7] and Britton and Walmsley [8] to estimate the charge density that would appear during tank filling and hence provide tank-filling voltage calculations for annular tanks. This extension of the basic calculation is necessary for the theoretical assessment of the influence of a "central conductor" on liquid surface voltage during tank filling.

3.2. Equations solved

The equations to be solved were set out by Asano [4], namely; Laplace's equation in the vapour space and Poisson's equation in the liquid space. The only difference between his analysis and the present one lies in the inner radial boundary condition. In a cylindrical tank it is $\partial\Phi/\partial r = 0$ at $r = 0$, whilst in an annular tank it is $\Phi = 0$ at $r = a_0$, where Φ is the potential, r is the radial coordinate and a_0 is the inner conductor radius. Throughout this paper we have solved the dimensionless equations formed by scaling Poisson's and Laplace's equations with the characteristic length scale $L^* = a$ (the tank radius) and the characteristic potential $\Phi^* = \rho a^2/\varepsilon_L\varepsilon_0$. The dimensionless equations thus formed are:

$$\nabla^2\Phi_v = 0 \tag{1}$$

for the potential, Φ_v, in the vapour space and

$$\nabla^2\Phi_L = 1 \tag{2}$$

for the potential, Φ_L, in the liquid space. The boundary conditions are that the potential is zero at the tank walls, including the surface of the inner conductor (i.e. for $z = 0$, $z = H$, $r = 1$ and $r = a_0$) and that $\Phi_v = \Phi_L$ and $d\Phi_v/dz = \varepsilon_L d\Phi_L/dz$ at the liquid surface (we assume no surface charge). The nomenclature is given in Appendix B.

3.3. Cases covered and the associated dimensionless voltages used to present the results

The real dimensional potentials, V_L and V_v, in the liquid and vapour spaces are given by:

$$V = \Phi\left(\rho a^2/\varepsilon_L\varepsilon_0\right) \tag{3}$$

where Φ represents one of the dimensionless potentials Φ_L or Φ_v and V represents V_L or V_v. Solutions to Equations (1) and (2) are discussed for two principle cases:

a) fixed charge density, ρ,
b) fixed total tank charge, Q.

Case a) is used for checking and comparing with Asano's results whilst Case b) is applicable to tank filling because, by the time the maximum potential is reached, the total charge in the tank usually has the constant value $Q = I\tau_{eff}$ (Britton and Walmsley [8]),[1] where I is the inlet streaming current and τ_{eff} is the effective relaxation time. The effective relaxation time is related to the relaxation time, τ, of the rest liquid by $\tau_{eff} = \tau\theta$ where θ is an empirical factor used to correct for departures from ideal relaxation behaviour. The relaxation time of the rest liquid is $\tau = \varepsilon_L\varepsilon_0/\kappa$ where κ is the liquid conductivity. Case b) can be divided into two subcases:

1) The central conductor behaves as a solid obstruction so liquid is confined to the space between the inner and outer conductors. In this case the volume occupied by the liquid and the charge is $v = \pi a^2(1^2 - a_0^2)z_s$.
2) The central conductor behaves like a perforated wall or mesh that sets the potential to zero at $r = a_0$ but allows the liquid and charge to occupy the entire interior of the tank including the space enclosed by the central conductor. The volume occupied by the liquid and the charge is then $v = \pi a^2 z_s$.

Subcase b1) represents top filling with a centrally located fill tube (see below) whilst Subcase b2) represents bottom filling with a centrally located perforated sampling tube. When the central conductor is small there is little difference between the two subcases

In Case a) it is most convenient to plot the results using the modified dimensionless potential $\Phi_a = V\varepsilon_0/\rho a^2 = \Phi/\varepsilon_L$ because this has the same dependence as the real voltage, V, on tank aspect ratio, relative liquid depth, relative inner conductor radius and liquid dielectric constant. In Case b) (tank filling) we employ the substitutions $Q = I\tau_{eff}$, $\tau_{eff} = \tau\theta$, $\tau = \varepsilon_L\varepsilon_0/\kappa$, $\rho = Q/v$ along with $v = \pi a^2(1^2 - a_0^2)z_s$ for Subcase b1) and $v = \pi a^2 z_s$ for Subcase b2). With these substitutions it is found that the alternative modified dimensionless potential $\Phi_b = V\pi\kappa a/I$ scales in the same way as the real voltage, V, and this is consequently the most convenient dimensionless voltage to use for presenting results. Subcase b1) gives $\Phi_{b1} = \Phi/[z_s(1 - a_0^2)]$ whilst Subcase b2) gives $\Phi_{b2} = \Phi/z_s$.

3.4. Solutions

When solving Equation (1) by separation of variables, the solutions take the form:

[1] It usually takes much longer than the effective charge relaxation time for the surface potential to reach a maximum. In this case, when the potential is at a maximum, the total charge takes the quoted fixed value if charge relaxation obeys Ohm's law.