Journal of Electrostatics 71 (2013) 746–753

Contents lists available at SciVerse ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

Energy, force and field strength in a system of two charged conducting balls

Vladimir A. Saranin*

Department of Physics, Glazov State Pedagogical Institute, Pervomayskaya St., 25, Glazov 427600, Russia

ARTICLE INFO

Article history:
Received 29 August 2012
Received in revised form
12 April 2013
Accepted 6 May 2013
Available online 23 May 2013

Keywords:
Conducting balls
Force of electrostatic interaction
Electric field strength

ABSTRACT

A calculation has been performed for the interaction force and the field strength in a system consisting of two charged conducting balls in three cases: 1) the balls are maintained at the preset potentials; 2) the balls are insulated and have fixed charges; 3) one of the balls has the preset potential, and the second one has the preset charge. Experimental measurements of forces in the first and third cases were performed and fair agreement with calculations was confirmed. A calculation method has been suggested, and the electric field strength on the nearest poles of the balls has been calculated for the three mentioned cases. A good agreement with the other authors' calculation results has been demonstrated.

1. Introduction

The charged conducting balls (spheres) are used in high-voltage arresters; the charged water drops interact in clouds; the charged particles interact in gas-dust plasma; after all, the problems of such ball interaction in different settings are included into almost all physics problem textbooks. All this makes the study of interaction and electric field of the charged conducting balls timelessly relevant. It was the subject of several research papers [1–6].

The fundamental work [1] suggests a ball interaction force calculation method based on the potential energy derivation, and the forces are calculated for the cases where the ball potentials are preset, and for the case where the ball charges are fixed. Also, in Ref. [1] a formula is obtained for calculation of the field strength on the nearest ball poles only for the case where the ball potentials are preset. In Ref. [2] the field force and strength in two conducting balls' system were calculated based on the solution of the Laplace equation in bispherical coordinates. In Ref. [3] the forces were calculated using the potential energy of the ball interaction only for the case of balls with the preset charges. In Ref. [4] the ball interaction force is calculated within the framework of the three problem settings: balls with constant charges; balls with the preset equal potentials; one ball having a preset charge, and another one having a preset potential. The same work contains the first-time electrostatic force measurements and comparison with the theory. In Ref. [5] integrates the results of [3,4] and other works of the author. In Ref. [6] the method of summing of forces acting on the electrostatic images in the balls was used. The resulting ball interaction force was calculated for the case of balls with the preset charges and equal radii based on this method.

The force and strength calculation method used in this work differs from the methods suggested in Refs. [2–6]. It enables performing the field force and strength calculations for all the three above mentioned problem settings within the common approach. This method is similar to the one used in Ref. [1] for the force calculation. However, the range of the problem settings is expanded as compared with problem [1] of the interaction of balls for one of which the potential is set, and for another the charge is set. In addition, the experimental measurements of the forces have been performed for this case. They validate the theoretical calculations (while there are no experimental results in Ref. [1]). The non-monotonic change of force has been found out as a function of distance in case of equal ball potentials. It has also been validated experimentally. The field strength has also been calculated in this work within the three above mentioned problem settings.

2. Method and example of force calculation

Three different problem settings can be outlined herewith: 1) during the interaction the balls remain disconnected from the power supply terminals in such a way that their potentials are fixed and known as U_1, U_2 (Fig. 1a); 2) both balls are pre-charged to the

* Tel.: +7 89128588865.
E-mail address: val-sar@yandex.ru.

0304-3886/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.elstat.2013.05.003

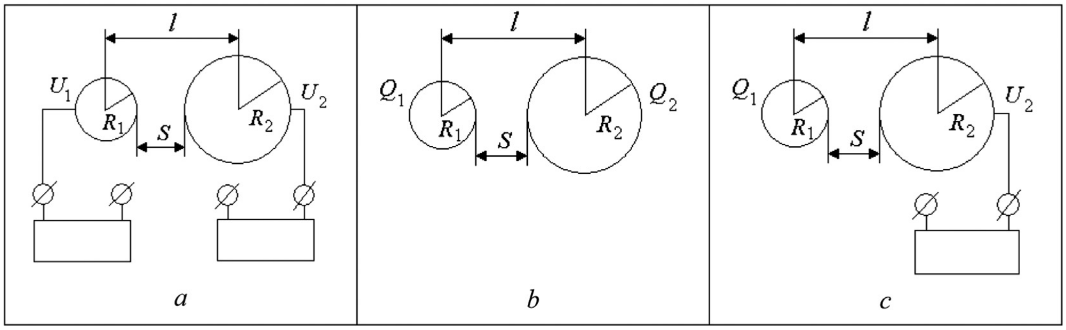

Fig. 1. Possible problem settings in a system of two balls.

known charges Q_1, Q_2, and then they are isolated (Fig. 1b); 3) one of the balls is pre-charged to charge Q_1, it is located near the second one and isolated. The second ball is maintained at the fixed potential U_2 (Fig. 1c).

1) If the permanent ball potentials are maintained, then the expression for the potential ball interaction energy can be written as [7]

$$W(l) = \frac{1}{2}\left(c_{11}U_1^2 + 2c_{12}U_1U_2 + c_{22}U_2^2\right), \tag{1}$$

where c_{ik} is the capacitance coefficients depending on the distance between the balls and on their radii, l is the distance between the ball centres. The explicit expressions of the capacitance coefficients for two conducting balls (spheres) are given in Ref.[7] as follows:

$$\begin{aligned} c_{11} &= \gamma \sinh\beta \sum_{n=1}^{\infty} [\gamma \sinh n\beta + \sinh(n-1)\beta]^{-1} \\ c_{12} &= -\frac{\gamma \sinh\beta}{r(1+\gamma)} \sum_{n=1}^{\infty} (\sinh n\beta)^{-1} \quad k = 1/4\pi\varepsilon_0, \\ c_{22} &= \gamma \sinh\beta \sum_{n=1}^{\infty} [\gamma \sinh n\beta + \gamma sh(n-1)\beta]^{-1} \end{aligned} \tag{2}$$

Here parameter β is associated with the distance between the ball centres l by a formula

$$\cosh\beta = \frac{x^2(1+\gamma)^2 - (1+\gamma)^2}{2\gamma}, \quad x = \frac{l}{R_1+R_2}, \quad \gamma = \frac{R_2}{R_1}. \tag{3}$$

Let us transform Eq. (1) as follows:

$$W(l) = \frac{U_1U_2}{2}(c_{11}/\alpha_U + 2c_{12} + c_{22}\alpha_U), \quad \alpha_U = \frac{U_2}{U_1}. \tag{4}$$

The ball interaction force can be determined as follows:

$$F_l(l) = \frac{\partial W(l)}{\partial l}. \tag{5}$$

Herewith, as is shown in Refs. [7], in this case of an open system, just sign "+" is used for differentiation. Let us consider variable x and calculate the capacitance coefficients in units of $(R_1 + R_2)/k$. Then the expression for the force can be set down as

$$F_x(x) = \frac{\partial W(x)}{\partial x} = \frac{U_1U_2}{2k}\left(c'_{11}/\alpha_U + 2c'_{12} + \alpha_U c'_{22}\right). \tag{6}$$

The accents denote the derivative with respect to x.

It is convenient to write all the magnitudes as dimensionless variables and express the force in units of the maximum force calculated in Coulomb approximation. It is the case where the ball charges are located in the ball centres, and any electrostatic image effects are neglected. Such a force is calculated when the balls contact, and it is equal to

$$F_{Cm}^{(U)} = \frac{|U_1U_2|R_1R_2}{k(R_1+R_2)^2}. \tag{7}$$

Then the ball interaction force is:

$$F_x^{(U)}(x) = F_{Cm}^{(U)}\frac{(1+\gamma)^2}{2\gamma}\left(c'_{11}/\alpha_U + 2c'_{12} + \alpha_U c'_{22}\right), \tag{8}$$

2) Let us assume now that the balls are charged, isolated and their charges are constant and equal to Q_1, Q_2 (Fig. 1b). Then the force is defined by the explicit formulae:

$$W(l) = \frac{1}{2}\left(Q_1^2 s_{11} + 2s_{12}Q_1Q_2 + Q_2^2 s_{22}\right), \quad F_l = -\frac{\partial W}{\partial l}, \tag{9}$$

where s_{ik} is the potential coefficients. As a result, the calculation of the force by this method is fairly tedious. This was the method used for force calculation in Refs. [4,5]. Another force calculation method is suggested below. It is simpler and universal, and can also be used for the field strength calculation.

As the inter-ball distance changes, the ball potentials change according to

$$\begin{aligned} U_1 &= Q_1 s_{11} + Q_2 s_{12} = \frac{kQ_1}{R_1+R_2}(\tilde{s}_{11} + \alpha_q \tilde{s}_{12}), \quad \alpha_q = \frac{Q_2}{Q_1} \\ U_2 &= Q_2 s_{22} + Q_1 s_{21} = \frac{kQ_2}{R_1+R_2}(\tilde{s}_{22} + \tilde{s}_{21}/\alpha_q), \end{aligned} \tag{10}$$

where $\tilde{s}_{ik}$ are the potential coefficients expressed in units of $k/(R_1 + R_2)$, which are associated with the capacitance one by the following formulae [7] (hereafter the sign "~" will be omitted):

$$s_{11} = \frac{c_{22}}{c_{11}c_{22} - c_{12}^2} \quad s_{12} = -\frac{c_{12}}{c_{11}c_{22} - c_{12}^2}, \quad s_{22} = \frac{s_{11}c_{11}}{c_{22}}, \quad s_{21} = s_{12}, \tag{11}$$

To obtain the expression for the force in this case it is sufficient just to replace U_1, U_2 with the right-hand terms of Eq. (10) in the right-hand terms of force Eq. (6) without any additional differentiation. This technique is new and results in efficiency of the suggested two balls' system force and strength calculation method. Then we shall obtain