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Electrical capacitance of dielectric coated metallic parallelepiped and closed cylinder isolated in free space

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ABSTRACT

This paper presents a method for the evaluation of the capacitance and the charge distribution of a dielectric coated metallic parallelepiped and a dielectric coated metallic hollow cylinder with the top and bottom cover plates using the method of moments (MoM) based on the pulse basis function and the point matching. Boundary conditions for the potential on the conductor surfaces and continuity of the normal component of the displacement density at the dielectric-free space interface is used to generate two integral equations. Two sets of simultaneous equations are formed from the two integral equations using the MoM. The total charge on the conductor surface is found from the solution for the set of simultaneous equations. The validity of the analysis has been justified by comparing the data on the capacitance available in the literature for metallic cube and hollow cylinder with top and bottom cover plates with the data on capacitance, computed by the present method for similar structures considering a very low dielectric constant as well as a very thin dielectric coating.

1. Introduction

During the erratic magnetic storm from the Sun, the amount of deposition of the charge particles on the surface of the exposed satellite body varies rapidly [1]. This variation in the rate of change of charge built up on the satellite surface results into a current pulse through the satellite body which in turn results into electromagnetic pulse causing the electromagnetic interference (EMI). The estimation of the equivalent circuit parameters is necessary for the study of EMI phenomenon in the spacecraft bodies. The capacitance is a very important circuit parameter needed for the analysis of electrostatic discharge (ESD), which causes EMI as mentioned above. The electrostatic modeling for the evaluation of the capacitance and the charge distribution of the finite metallic bodies with different geometrical shapes such as rectangular plate, cylinder, truncated cone, paraboloid, hemisphere, cube, corner etc. which are widely used in the orbiting spacecraft are discussed in [2–16]. The metallic bodies of a spacecraft are covered with dielectric material for thermal control. Thus, the evaluation of the capacitance and the charge distribution of these metallic bodies covered with dielectric coating is of practical interest. In presence of a dielectric coating on the metallic surface, the modeling becomes more complex, since in this case one has not only to solve for the charges on the conducting surface, but also account for the polarization effect on the dielectric coating. A variety of computational techniques such as MoM [2–8], variational iteration method (VIM) [19,22,23], and homotopy perturbation method (HPM) [20,21], can be used for solving the problem as discussed above. The analysis of a metallic cube is presented in [3,12], using the MoM. The MoM analysis of a hollow metallic cylinder with top and bottom cover plates is presented in [13]. Thus, the evaluation of the capacitance of a dielectric coated metallic parallelepiped and a hollow cylinder with top and bottom cover plates are of interest, since the same are not available in the literature to the best of the authors' knowledge.

In this paper, the evaluation of the capacitances and the charge distribution of the structures in the form of metallic parallelepiped and hollow cylinder with top and bottom cover plates, with a uniform dielectric coating are presented using the MoM.

2. Analysis

2.1. Dielectric coated parallelepiped

A dielectric coated parallelepiped with the dimensions $2a \times 2b \times 2c$ with the uniform coating of thickness Δ, on the outer surface, is shown in Fig. 1. In order to apply the MoM, each of the metallic and dielectric surface is divided into $M_l \times N_l$; $l = 1, 2, 3, \ldots,$

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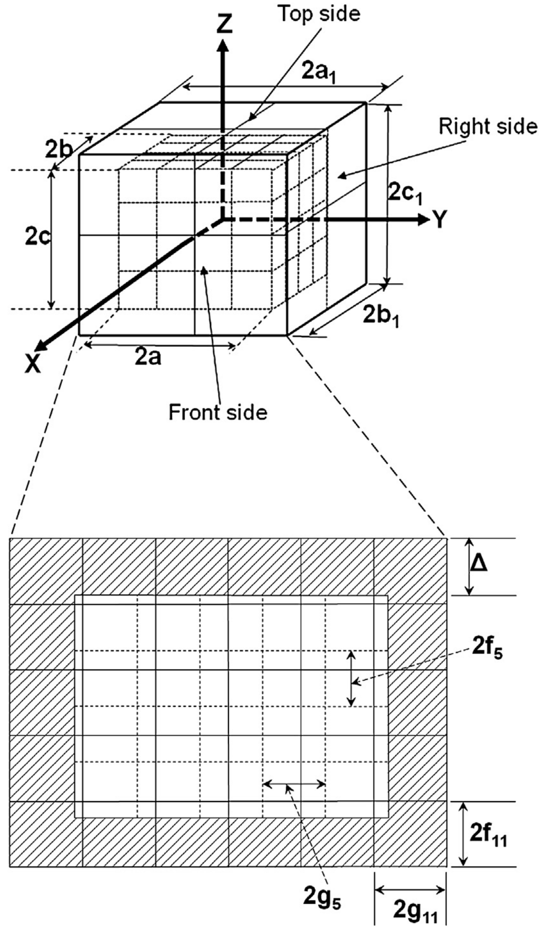

Fig. 1. Dielectric coated parallelepiped with expanded front side.

12. Six metallic surfaces of the parallelepiped; top, bottom, right, left, front and back are divided into subsections of the size $2g_i \times 2f_i$; $i = 1, 2, \ldots, 6$, respectively. The dielectric coating on all the six metallic surfaces; top, bottom, right, left, front and back are divided into subsections of the size $2g_j \times 2f_j$; $j = 7, 8, \ldots, 12$, respectively. It is assumed that the charge is uniformly distributed on each surface of the parallelepiped. However, in order to calculate the potential, it is assumed that the total charge in a particular sub-area is concentrated at its centre. The potential at any arbitrary point due to the charge distribution on the conductor and the dielectric surfaces is of the form [2,5,14],

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{12}\left[\int_{s^i}\frac{\sigma_i(\vec{r}'_i)}{|\vec{r}-\vec{r}'_i|}\right] \tag{1}$$

In order to find the capacitance of the structure shown in Fig. 1, the knowledge of total unknown charge distribution on the metallic surfaces and the value of potential, which is equal on the metallic surfaces, should be known. Assuming the value of the potential on the metallic surfaces as V_i; $i = 1, 2, \ldots, 6$; the following integral equations are obtained [2,5,14],

$$V_i(\vec{r}_i) = \frac{1}{4\pi\varepsilon_0}\left[\sum_{j=1}^{12}\int_{s^j}\frac{\sigma_j(\vec{r}'_i)\,\mathrm{d}s}{|\vec{r}_i-\vec{r}'_i|}\right] \tag{2}$$

For simplicity, the following assumptions are made,

$$k_n = \sum_{i=1}^{n} M_i N_i; \quad n = 1, 2, 3, \ldots, 12 \tag{3}$$

The unknown charge distribution over six conductors and six dielectric faces of the parallelepiped appearing in Eq. (1) are expressed in terms of known basis functions as [14],

$$\sigma_i(\vec{r}'_i) = \sum_{n=k_{i-1}+1}^{k_i}\alpha_{in} f_n; \quad i = 1, 2, \ldots, 12 \tag{4}$$

where, $k_0 = 0$ and f_n is the pulse function given by,

$$f_n = \begin{cases} 1, & \text{on } \Delta s_n \\ 0, & \text{on any other } \Delta s_m \end{cases}$$

The X, Y and Z-components of electric flux densities at any point due to charge distribution on the conductor surfaces of front–back, left–right and top–bottom faces of the parallelepiped, respectively, are given by [5],

$$\vec{D}_X = -\vec{u}_X\varepsilon\frac{\partial\Phi}{\partial X}; \quad \vec{D}_Y = -\vec{u}_Y\varepsilon\frac{\partial\Phi}{\partial Y}; \quad \vec{D}_Z = -\vec{u}_Z\varepsilon\frac{\partial\Phi}{\partial Z} \tag{5}$$

Applying the boundary condition of the normal component of flux density at the air–dielectric interface,

$$D_{n_b}\ (\text{air}) + D_{n_{fb}}\ (\text{air}) = D_{n_{fb}}\ (\text{dielectric}) \tag{6}$$

In Eq. (6), D_{n_b} represents normal component of the electric flux density in the air–dielectric interface due to bound charge on the dielectric surface, $D_{n_{fb}}$ represents the normal component of the electric flux density due to charges on the conductor surface. Substituting Eq. (4) in Eq. (2) and matching the boundary conditions for the potential at the centre point of each subsection for the conductor and the dielectric surfaces and following the procedure of [5], the equation reduces to the form,

$$\begin{aligned} V_i &= \sum_{j=1}^{12}\left[\sum_{n=k_{j-1}+1}^{k_j}\alpha_{jn} l_{i,jmn}\right], \quad i = 1, 2, 3, 4, 5, 6; \\ m &= k_{i-1}+1, k_{i-1,}+2, \ldots, k_i; \;\; k_0 = 0 \end{aligned} \tag{7}$$

Substituting Eqs. (5) and (6) in Eq. (2) and matching the boundary condition at dielectric-air interface, and following the procedure of [5], the following equations are obtained, for the top surface,

$$\begin{aligned} 0 = {} & \sum_{n=1}^{k_1}\alpha_{1n} l_{7,1mn} + \cdots + \sum_{n=k_5+1}^{k_6}\alpha_{6n} l_{7,6mn} \\ & -\frac{1}{\varepsilon_0(\varepsilon_r-1)}\sum_{n=k_6+1}^{k_7}\alpha_{7n} l_{7,7mn} + \sum_{n=k_7+1}^{k_8}\alpha_{7n} l_{7,8mn} + \cdots \\ & + \sum_{n=k_{11}+1}^{k_{12}}\alpha_{7n} l_{7,12mn}; \; m = k_6+1, k_6+2, \ldots, k_7 \end{aligned} \tag{8}$$

Similar equations are obtained for the right, left, front and back surfaces. The set of simultaneous equations resulting from Eqs. (7) and (8) lead to matrix equation of the form,