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Effective permittivity of a fiber-reinforced composite with transversely isotropic constituents

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ABSTRACT

Simple closed-form expressions for effective permittivity of fiber-reinforced composites with transversely isotropic constituents are found using asymptotic homogenization. Circular cylindrical fibers are distributed in a square array. The analysis considers four orientations of constituents transverse symmetry axis relative to fibers direction. Local problems defined on a periodic square unit cell are solved by means of complex potential theory using Weierstrassian and Natanzon's functions, assuming that the contrast of permittivities is small. Derived closed-form formulas are compared with finite element calculations and, in the isotropic case, with standard mixture rules and classical bounds, obtaining excellent results even when contrast is large.

1. Introduction

Composite materials have been studied theoretically for a long time [12,38,39]. These studies have led to mixing rules relating the macroscopic dielectric properties of heterogeneous media to those of their constituent phases and the internal structure of the mixture [44].

Being able to characterize a heterogeneous material by means of its effective properties is essential to researchers and industries in fields as varied as automotive, construction, biomedical, sports, aerospace, remote sensing, etc. Besides, the rising popularity of microwaves for drying [17,50], material processing [29] and sensing [31,34] make determination of effective properties of composites a hot topic. Several techniques and numerous studies have been developed to predict the behavior of composite materials. Willis [52] classified them into four main categories: asymptotic, self-consistent, variational, and modeling methods. A summary of most important macroscopic mixture relations can be found in Tinga [49].

Among these techniques, the effective medium theory (EMT) is considered as the most powerful approach to estimate the effective properties for the composite systems, such as cosmic dusts, aerosols, and porous media [30]. The simplest approximation is the one used by Wiener, it takes simple arithmetic and harmonic averages and provides an upper and a lower bound for the effective permittivity [9]. This approach does not make any assumption on inclusion shape, however, it is important. A more rigorous bounds were found by Hashin and Shtrikman [16] using variational principles. Although the original derivation was done for magnetic susceptibility, they can be applied to permittivity, conductivity and elastic problems. One of the oldest and most popular EMT is the Maxwell-Garnett (MG) mixing rule that was developed for optical properties of a medium [27]. The assumption that separation between inclusions is large, limits the applicability of MG to dilute systems with a low volume fraction. Corrections to the MG theory have been proposed before [2,3,13]. An important MG theory improvement came with another popular mixing rule, the Bruggeman approximation (B), that was developed for the effective electrical properties of a composite system [7]. Another MG and B

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theory improvement was given by Looyenga [24] (L), where predictions agree well with experiments [17,19,46,53]. Besides the above mentioned techniques, there are many other methods for finding effective dielectric behavior of composites [5,11,15,18,48], for instance.

The techniques mentioned above have been used for a long time for predicting the behavior of an isotropic composite with isotropic phases. However, this estimations are not enough when predicting effective properties of anisotropic materials with anisotropic phases. Levy and Stroud [23] used the MG theory to find the effective dielectric behavior for a medium of anisotropic inclusions embedded in an isotropic host. Sushko [47] used an approach, based upon the notion of macroscopic compact groups of particles, to derive dielectric mixing rules for macroscopically homogeneous and isotropic multicomponent mixtures of anisotropic inhomogeneous dielectric particles. Sihvola [45] studied isotropic inclusions in an anisotropic medium. Bianisotropic host media has been considered as well [51] into the framework of MG and B theory.

As far as authors know, there are few previous works on modeling different axis of transverse symmetry orientation of constituents. Kar-Gupta and Venkatesh [20] studied the electromechanical behavior, numerically, of a 1–3 composite by changing the polarization direction of matrix. Sakthivel and Arockiarajan [42] proposed an analytical model based on parallel and series theory for 1–3–2 piezoelectric composite. A number of methods have been applied to composites with different relative orientation of the material symmetry axis of each component [21,22,32,33,42], etc. The antiparallel case analyzed in Kar-Gupta and Venkatesh [21] can be considered as a particular case of the formulas developed in Sabina et al. [41] and in Bravo-Castillero et al. [6]. In those papers, closed-form expressions for effective coefficients of a piezoelectric fiber-reinforced composite are found using the asymptotic homogenization method (AHM). This mathematical technique is used for studying partial differential equations with rapidly oscillating coefficients [1,4,43]. Composites are subjected to fields which vary on a lengthscale, due to their microstructures. The AHM replaces the rapidly oscillating coefficients of the differential equation with constant effective coefficients that depend on the solution of the so-called local problems.

Within this paper, it is proposed an application of the AHM to determine the effective dielectric behavior of a fiber reinforced composite of circular cylinders distributed in a square periodic arrangement with material symmetry axis orthogonal to the fiber direction. In Section 2 the problem at hand is established as well as relevant definitions and notation, finally, some outstanding equations for AHM are presented. The way local problems are solved is presented in Section 3, aside from formulas for effective coefficients for each considered case. Numerical calculations and comparison of obtained formulas can be found in Section 4. Empiric intervals of validity are found and they are explicitly written in this section too. Finally, some concluding remarks are expressed in Section 5.

2. Statement of the problem

An anisotropic fiber-reinforced dielectric material, occupying a region Ω in a tridimensional Euclidian space, is considered here. Parallel fibers, with circular cross-section, are aligned, without overlapping, periodically into a square symmetry, their direction is taken as the Ox_3-axis of a cartesian reference system. Fibers and matrix have transversely isotropic dielectric properties. Within this paper we study four possible orientations of the axis of transverse symmetry (ATS) of each component: i) P3. ATS of each component coincide with the Ox_3-axis; ii) O2. The ATS of matrix is parallel to the Ox_1-axis; the ATS of fiber, to the Ox_3-axis; iii) P1. Matrix and fiber ATS are parallel to the Ox_1-axis; iv) O3. Matrix and fiber ATS

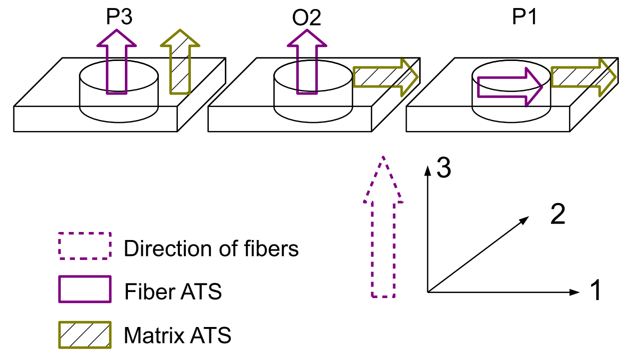

Fig. 1. Possible ATS orientation: i) P3. Both ATS are parallel to Ox_3-axis, ii) O2. Both ATS are orthogonal to Ox_2-axis, iii) P1. Both ATS are parallel to Ox_1-axis, iv) O3. (Not shown) with a suitable redefinition of parameter α_2 it becomes a particular case of P1 case.

are orthogonal to each other and with the fibers direction. With a suitable redefinition of parameter α_2 (defined in Eq. (13)), this is a particular case of P1 case (see Fig. 1).

Overall properties of the periodic media described above are sought by means of the asymptotic homogenization method (AHM) [10]. The periodic unit cell S is taken as a square in the y_1y_2-plane, where $\mathbf{y} = \mathbf{x}/\varepsilon$ is the local (fast) coordinate related to $\mathbf{x}$, the macroscopic (slow) variable by means of the small parameter ε. The periodic unit cell consists of two phases: a single fiber (S_1) and the matrix that surrounds it (S_2), each of volume fraction V_1 and V_2, respectively. In addition, they are such that $V_1 + V_2 = 1$, $S_1 \cup S_2 = S$, $S_1 \cap S_2 = \varnothing$. The interface between fibers and matrix is denoted by Γ (Fig. 2).

Here the electric displacement $\mathbf{D}(\mathbf{x})$ is linearly related to the electric field $\mathbf{E}(\mathbf{x})$ through the second rank permittivity tensor, $\kappa_{ij}(\mathbf{x})$ by:

$$D_i(\mathbf{x}) = \kappa_{ij}(\mathbf{x})E_j(\mathbf{x}), \tag{1}$$

where Einstein's summation convention over repeated indices is assumed. Latin indices take values in {1, 2, 3}, while the Greek indices in {1, 2}. In what follows, unless otherwise indicated, this notation is assumed on repeated indices.

The permittivity of matrix and fibers is represented by $\kappa_{ij}^{(1)}$ and $\kappa_{ij}^{(2)}$, respectively. They are assumed to be transversely isotropic second rank tensors.

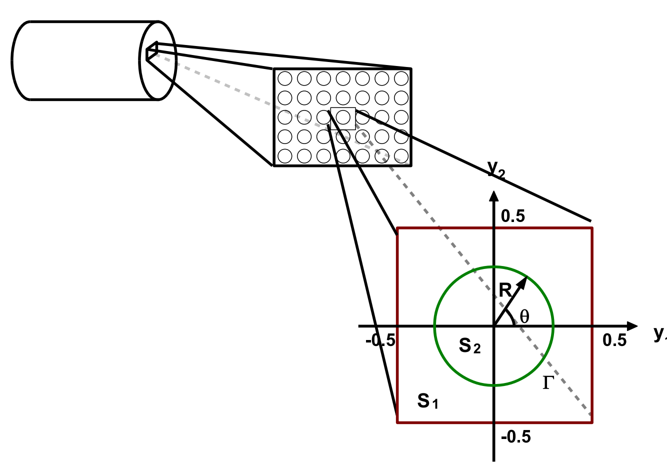

Fig. 2. Schematic view of periodic fiber-reinforced composite, the coordinate system y_1y_2 and its periodic unit cell, it comprises a single fiber S_2 and the matrix S_1 that surrounds it. $S = S_1 \cup S_2$, $\Gamma = \partial S_2$ and $S_1 \cap S_2 = \varnothing$