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# Shape reconstruction of mixed type void flaws using Born inversion method

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ABSTRACT

We study an approximate technique for determining the shapes of mixed type void flaws in elastic media from knowledge of the ultrasonic scattering amplitudes. It is well known that the technique is highly effective under weak scattering conditions. In the paper, two cement paste cylindrical specimens with mixed type void flaws are prepared and ultrasonic measurements are carried out by experimental means. The measurement area is restricted in the plane perpendicular to the axis of cylindrical specimen. The measured wave data are fed into the approximate technique formula—the Born inversion method and cross-sectional image is obtained. We find that good results have been obtained for strong scattering void flaws such as mixed type void flaws in cement paste cylindrical specimens.



## 1. Introduction

The American society for NDT defines it as an examination of an object or material in a manner that will not impair its future usefulness. Nondestructive testing is used extensively throughout the industry for quantitative assessment and detection of defects in engineering materials. Visual images that describe the location, size and shape of embedded damage or flaws provide a direct way to help engineers evaluate the condition of the concrete structures. In recent years, since the size, shape and location of defects are a fundamental information to estimate the residual life of the structural component and the service life of the component, many studies about the detection and characterization of defects have been developed [1,2]. There are many cylindrical-shaped civil structures like a bridge pier. In this article, the objects are cylindrical cement paste specimens which have the mixed type void flaws. When the specimen is inspected by ultrasonic waves, the access point of transducer is limited to the surface of the cylinder. The approximate technique—three-dimensional inverse scattering method [3] is modified to the convenient form for cylindrical structures. This method is based on the elastodynamic Born [4,5] inverse scattering method. In recent years, the Born inversion method is used in many fields, such as the image reconstruction algorithm of the flaw [6,7], subspace techniques [8] and electromagnetic (EM) fields [9]. At the same time, it is also applicable to the cement-based material [10], since the low frequency component of measured wave data plays key roles in the method. In experimental measurement of this article, two cement paste cylinders with mixed type void flaw models are prepared. Moving the transducer along the measurement plane of the cylinder, the scattering amplitudes obtained from the measurement are fed into the Born inversion and cross-sectional image of the cylinder is obtained by the approximate method. We reconstruct the shape of the mixed type void defect by the approximate technique. It is verified that the Born inversion method is suitable for strong scattering materials under certain circumstance.

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## 2. Approximate technique

### 2.1. Integral representation for scattered field

One type of integral representation for scattered fields from flaws is summarized for the latter use with approximation. The type is volume type and it is prepared for the Born inversion. In the three-dimensional elastic material $D$ with a flaw region $D^C$ as shown in Fig. 1, let $C_{ijkl}$ and $\rho$ be elastic modulus and mass density of the host matrix $D \setminus D^C$. The elastic modulus and mass density in the flaw region $D^C$ are denoted by $C_{ijkl} + \delta C_{ijkl}$ and $\rho + \delta\rho$. Then the governing equation of motion for the displacement $u$ in the whole material $D$ can be written as

$$C_{ijkl}u_{k,lj} + \rho\omega^2 u_i = -[\delta\rho\omega^2 u_i \Gamma(x) + \delta C_{ijkl}u_{k,lj}\Gamma(x) + \delta C_{ijkl}u_{k,l}\Gamma(x)_{,j}] \tag{1}$$

where $\Gamma(x)$ is the characteristic function of the flaw region $D^C$ defined by

$$\Gamma(x) = \begin{cases} 1 & \text{for } x \in D^C, \\ 0 & \text{for } x \in D \setminus D^C. \end{cases} \tag{2}$$

From Eq. (1), the scattered wave field $u_m^{sc}(y)$ from flaw has the following integral representation[11]

$$u_m^{sc}(y) = \int_D G_{im}(x,y)\{\delta\rho\omega^2 u_i(x)\Gamma(x) + \delta C_{ijkl}u_{k,lj}(x)\Gamma(x) + \delta C_{ijkl}u_{k,l}(x)\Gamma_{,j}(x)\}dV \tag{3}$$

where $G_{im}(x,y)$ is the fundamental solution for three-dimensional elastodynamics and expressed as

$$G_{im}(x,y) = \frac{1}{4\pi\mu}\left[\frac{e^{ik_T r}}{r}\delta_{im} + \frac{1}{k_T^2}\frac{\partial}{\partial x_i}\frac{\partial}{x_m}\left\{\frac{e^{ik_T r}}{r} - \frac{e^{ik_L r}}{r}\right\}\right] \tag{4}$$

The details of the transformation needed to obtain Eq. (5) from Eqs. (2) and (3) are given in Appendix A. The scattered field in Eq. (3) reduces to more amenable form from the nature of $\Gamma(x)$ in Eq. (2).

$$u_m^{sc}(y) = \int_D \Gamma(x)\{\delta\rho\omega^2 u_i(x)G_{im}(x,y) - \delta C_{ijkl}u_{k,l}(x)G_{im,j}(x,y)\}dV \tag{5}$$

### 2.2. Equivalent scattering source

The scattered wave field $u_m^{sc}(y)$ has been represented by the type of integral in Eq. (5). The integral representation for the scattered field $u_m^{sc}(y)$ is unified in a form by introducing the equivalent source $q_i(x)$ as

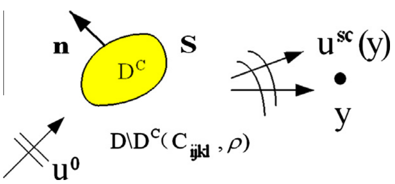


**Fig. 1.** Schematic of flaw's near-field scattering.

$$u_m^{sc}(y) = \int_D G_{im}(x,y)q_i(x)dV \tag{6}$$

where the fundamental solution $G_{im}(x,y)$ has been given in Eq. (4) and the equivalent source $q_i(x)$ is expressed in the following way. For volume representation in Eq. (5), the equivalent source $q_i(x)$ in Eq. (6) reduces to

$$q_i(x) = \Gamma(x)\{\delta\rho\omega^2 u_i(x) - \delta C_{ijkl}u_{k,l}(x)\partial/\partial x_j\} \tag{7}$$

where $\Gamma(x)$ is the characteristic function defined in Eq. (2).

### 2.3. Far-field representation

In the NDE application, the field point $\boldsymbol{y}$ is usually far from the surface of scatterer. Therefore we introduce the far-field approximation in the following sense

$$|y - x| = |y| - \hat{y}\cdot x \tag{8}$$

where $y = y/|y|$ is the unit vector point to the field point $y$ from the origin of coordinate system. It is to be remarked that the coordinate origin is not necessary at the center of scatterer. It may be located off from the scatterer as shown in Fig. 2.

Introduction of the far-field approximation in Eq. (8) into the expression of fundamental solution in Eq. (4) leads to the far-field approximation of the fundamental solution

$$G_{im}(x,y) = \frac{1}{\mu}[D(\mathbf{k_L}|\mathbf{y}|)\kappa^2\hat{y}_i\hat{y}_m e^{-ik_L\hat{y}\cdot x} + D(\mathbf{k_T}|\mathbf{y}|)(\delta_{im} - \hat{y}_i\hat{y}_m)e^{-ik_T\hat{y}\cdot x}] \tag{9}$$

where $D(z) = e^{iz}/(4\pi z)$ and $\kappa = k_L/k_T$. From Eqs. (5) and (9), the expression of the scattered far-field reduces to

$$u_m^{sc:far}(y) == D(\mathbf{k_L}|\mathbf{y}|)A_m(\hat{y}) + D(\mathbf{k_T}|\mathbf{y}|)B_m(\hat{y}) \tag{10}$$

In Eq. (10), $A_m(\hat{y})$ is the longitudinal scattering amplitude and $B_m(\hat{y})$ is the transverse scattering amplitude and they have the form

$$A_m(\hat{y}) = \frac{\kappa^2}{\mu}\hat{y}_i\hat{y}_m\int_D q_i(x)e^{-ik_L\hat{y}\cdot x}dV \tag{11}$$

$$B_m(\hat{y}) = \frac{1}{\mu}(\delta_{im} - \hat{y}_i\hat{y}_m)\int_D q_i(x)e^{-ik_T\hat{y}\cdot x}dV \tag{12}$$

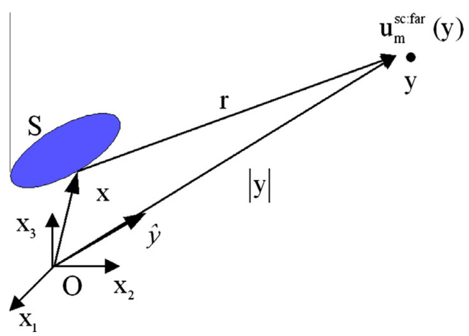


**Fig. 2.** Schematic of flaw's far-field scattering.