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An application-oriented mathematical meta-model of measurement

Roman Z. Morawski *

Warsaw University of Technology, Faculty of Electronics and Information Technology, Institute of Radioelectronics, Nowowiejska 15/19, 00-665 Warsaw, Poland

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ABSTRACT

The need for broader use of concepts of modelling in the development of a conceptual basis for measurement science is ascertained. A brief review of basic concepts of mathematical modelling is provided, and a class of models, most frequently used in measurement science, is characterised. A meta-model of measurement, unifying many existing methodologies of mathematical modelling in measurement science, is proposed. Its applicability is illustrated with a set of diversified examples.

1. Introduction

The need for a general model of measurement – called hereinafter "measurement meta-model" to emphasise that it should be able to encompass the models of specific measurement processes and systems – seems to be widely recognised. Various attempts, aimed at designing such a meta-model, have been undertaken since the second half of the XIXth century – their review may be found in the 2003 paper *Epistemology of measurement* [1]. The history of the *International vocabulary of basic and general terms in metrology* (*cf.* its consecutive versions *VIM1* [2], *VIM2* [3] and *VIM3* [4]), and especially the discussions related to its recent version, have also demonstrated the growing importance of the concepts of modelling – in particular of mathematical modelling – for the development of a conceptual basis of measurement science.

The measurement meta-model should be general enough to encompass all or at least vast majority of measuring systems, met in practice, and – at the same time – it should be specific enough to have higher explanatory power with respect to those systems than any general methodology for abstract modelling of real-world entities. It should describe the structure and behaviour of a generic measuring system, together with the procedure necessary for its proper operation. It is also reasonable to expect it to be compatible with the conceptual basis of *VIM3*. In this paper, being an extended and updated version of a conference paper [5], an attempt has been made to develop a meta-model satisfying all those requirements at the application level only, *i.e.* leaving entirely open fundamental epistemological issues.

The paper is structured as follows: first, in Sections 2 and 3, a general introduction to mathematical modelling and model validation is given; next, in Section 4, the proposed meta-model is outlined; finally, in Sections 5 and 6, its applicability for defining basic concepts of metrology and for designing specific measuring systems is demonstrated.

2. Basic concepts of mathematical modelling

There is no generally accepted definition of truth in epistemology because of the fundamental uncertainty about the relationship between reality and ideas assumed to describe it. The experts' views on this issue diverge and depend mainly on their epistemological orientation; there is no logical or empirical means to prove any of those views. The same applies to the concept of modelling. Therefore, we assume here that it is a primary concept

* Tel.: +48 22 234 7721.
E-mail address: r.morawski@ire.pw.edu.pl

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which cannot be formally defined, although it may be up to certain extent explained by its relations to other fundamental concepts of philosophical and scientific language, as well as by examples of its application. Mathematical modelling is, roughly speaking, modelling by means of mathematical structures, such as variables and equations. It is always preceded by semantic modelling (*cf.* Fig. 1), called also verbal or linguistic modelling, which consists in describing reality by means of meaningful statements of a natural language.

The problem of mathematical modelling may be considered from various perspectives or at various levels of generality; the following seem to be most widely recognised:

- ontological and epistemological justification of modelling;
- semantic modelling;
- mathematical meta-modelling of classes of systems;
- mathematical modelling of specific systems.

The scope of considerations presented in this paper is limited to two last perspectives/levels.

Let's assume that an entity to be modelled – for brevity, called "system" hereinafter – is an object, or a phenomenon, or an event, or…of physical, or chemical, or biological, or psychological, or sociological, or economical, …, or mixed nature. Its mathematical model is its description – composed of entities such as numbers, variables, sets, equations, functions and operators – which enables one to infer about the properties and/or behaviour of the modelled system under various conditions. The identification of the system itself, *i.e.*, the distinction between the system and the surrounding environment, is not objectively given, but is already part of the modelling process. It is possible that even in trivial situations, a mathematical model does not carry full information on the modelled system. So, a general criterion to evaluate the quality of a mathematical model refers to the trade-off between simplicity (plausibly inversely related to modelling costs) and informativeness: a mathematical model should be as simple as possible, but sufficiently informative for its target application.

Example 1. The simplest mathematical model of a resistor – $u = Ri$, where u is voltage, i is current, and R is resistance – is sufficient for the analysis of its static or low-frequency behaviour provided it is a high-precision resistor. For the analysis of the static or low-frequency behaviour of a low-precision resistor, an algebraic nonlinear model may turn out to be necessary. For an analysis of its high-frequency behaviour, a model having the form of a system of nonlinear ordinary differential equations should be used. A model reflecting only electrical phenomena in a resistor may be sufficient for the functional design of an electrical filter, but not sufficient for its technological design since thermal phenomena and special relations should be taken into account at this stage of its development. The technological design of the filter may also require randomisation of some parameters of the model when a kind of robustness of the designed filter with respect to technological tolerances is expected.

The process of mathematical modelling typically starts with an informal description of the modelled system in terms of its features which are considered to be important for the given application. Next, this description is translated into a more formal language of quantities which are idealised features of the system, obtained by means of abstraction. Here, the proper definition of a mathematical model begins, being generally an iterative procedure, composed of two fundamental operations (*cf.* Fig. 2):

- structural identification, *i.e.*, the selection of a structure for the model (most frequently, a type of function or equation, *e.g.*, a linear algebraic equation);
- parametric identification, *i.e.*, the estimation of the model parameters (*e.g.*, the coefficients of the linear algebraic equation).

The first operation hardly can be algorithmised: the choice of the model structure is usually based on some intuitive premises, anterior experience, and trial-and-error steps. On the other hand, the second operation is subject of advanced algorithmisation. By principle, a model provides only an approximate description of the properties and behaviour of the modelled system. Both model structure and parameters are affected by the already mentioned trade-off between simplicity and informativeness. The model is subject to structural inadequacies resulting from the limitations of the available knowledge on the modelled system, from neglecting some factors in the choice of the quantities (input, output and influence quantities) modelling the system, or from the inappropriate specification of such quantities, or from the inappropriate choice of the equations modelling the relationships among those quantities. It is also subject to inaccuracies in the parameter estimates due

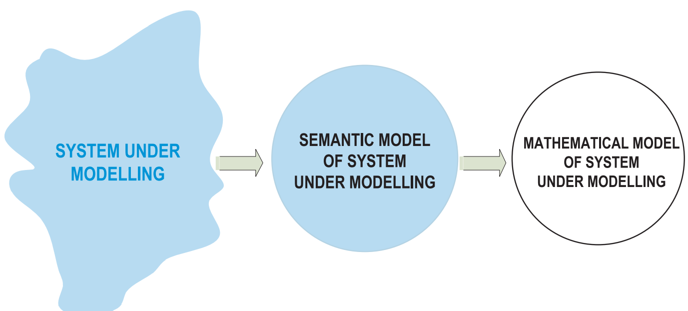

Fig. 1. General scheme of modelling reality.