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Technical note

A fast calibration-independent method for complex permittivity determination at microwave frequencies

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ABSTRACT

A transmission/reflection microwave method based on uncalibrated *S*-parameter measurements for complex permittivity determination of dielectric materials is presented. There are three main advantages of the proposed method. First, the measurements are performed without the need of any calibration standards. Second, it does not require any additional dielectric sample with different thickness; two uncalibrated measurements are required: (i) with a sample filled waveguide and (ii) with an empty waveguide. Third, it does not need a precise location or precise shifting distance of the sample inside the waveguide. The method is iterative needing an initial guess to start the mathematical calculations, and high measurement accuracy can be expected. The method is validated by complex uncalibrated *S*-parameter measurements at X-band frequencies of low-loss samples (Teflon, Celotex and Duroplex) fitted into a waveguide section.

1. Introduction

The study of dielectric properties of materials finds its applications in various fields. Applications in telecommunications and microwave for industry require accurate knowledge of the complex permittivity of used materials. However, the rectangular waveguides are widely used for broadband microwave characterization [1–4]. We present an iterative method based on the S_{ij} scattering parameters of the rectangular waveguide. The S_{ij} parameters are measured in transmission/reflection (T/R) without calibration of the Vector Network Analyzer (VNA). The experimental technique involves placing the sample of material under test (MUT) inside the WR90 rectangular waveguide and measuring the S_{ij} parameters. A second uncalibrated measurement of the empty waveguide is necessary to find iteratively the complex relative permittivity of the MUT. The mathematical approach of the proposed method is rigorous without any approximation, taking into account all reflections of the electromagnetic wave on the both sides of the sample sections. The complex permittivity of materials, ε_r^*, can be determined using these S_{ij} parameters. This calibration-independent technique is very important and attractive for many reasons: (1) requirement of some calibration manipulations before measurements is eliminated; we note that these standards manipulations cause an inevitable errors due to their imperfections; (2) using one sample leads to accurate values of ε_r^* than using two identical samples with different lengths; the thickness uncertainty is thus reduced; and (3) this technique does not need a precise location or precise shifting distance of the sample inside the waveguide. The method was applied to the determination of the complex relative permittivity of some solid dielectric materials (Teflon, Celotex and Duroplex) at X-band frequencies. The experimental results are compared with the obtained data by applying the same method on the calibrated scattering parameters.

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2. Theory

We consider the measurement setup shown in Fig. 1. The dielectric sample under test with length d is located in the mid of the waveguide. The sample is machined to the same rectangular waveguide section. It is assumed non-magnetic ($\mu_r^* = 1$), symmetric and isotropic. The two plans X and Y in Fig. 1 are considered as transitions between the sample/air interfaces and the rest measurement setup. These ports include air line regions, source and load match errors, tracking (frequency) errors, hardware imperfections of VNA, waveguide metallic losses, effects of wires carrying interconnections, coupling to higher order modes [1,5], etc. We also assume that only the dominant mode TE_{10} is present inside the waveguide sample portion.

The mathematical analysis is essentially based on the wave cascading matrix (WCM) [6,7]; we calculate the corresponding transmission matrices according to following equation:

$$M_i = \frac{1}{S_{21_i}} \begin{pmatrix} S_{12_i}S_{21_i} - S_{11_i}S_{22_i} & S_{11_i} \\ -S_{22_i} & 1 \end{pmatrix} \quad i = 1 \text{ or } 2 \tag{1}$$

M_1 corresponds to the uncalibrated S-parameter measurement that the waveguide is partially filled with a standard dielectric that its complex relative permittivity ε_{r1}^* is well known at the studied frequency band. M_2 corresponds to the uncalibrated S-parameter measurement that the waveguide is partially filled with the MUT that its complex relative permittivity ε_{r2}^* is unknown. M_1 and M_2 can also be written as a product of five matrices:

$$\begin{cases} M_1 = T_X \cdot T_{ref1} \cdot T_1 \cdot T_{ref1}^{-1} \cdot T_Y \\ M_2 = T_X \cdot T_{ref2} \cdot T_2 \cdot T_{ref2}^{-1} \cdot T_Y \end{cases} \tag{2}$$

We denote the two port WCM matrices (T_X,T_Y), (T_{ref1},T_{ref2}), and (T_1,T_2), respectively, for modeling the X and Y ports, the impedance jumps Air/Sample/Air at X and Y, and the ideal lines with length d. T_X and T_Y are assumed unchanged during measurements. $T_{ref1,2}$ and $T_{1,2}$ may be written in the following forms:

$$T_{refi} = \begin{pmatrix} \frac{1}{1-\Gamma_i} & \frac{\Gamma_i}{1-\Gamma_i} \\ \frac{\Gamma_i}{1-\Gamma_i} & \frac{1}{1-\Gamma_i} \end{pmatrix} \tag{3}$$

$$\Gamma_i = \frac{\gamma_0 - \gamma_i}{\gamma_0 + \gamma_i} (\mu_r^* = 1) \tag{4}$$

and

$$T_i = \begin{pmatrix} e^{-\gamma_i d} & 0 \\ 0 & e^{\gamma_i d} \end{pmatrix} \quad \text{with } i = 1 or 2 \tag{5}$$

where

$$\gamma_0 = j\frac{2\pi}{\lambda_0}\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}; \gamma_i = j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_{ri}^*\mu_r^* - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

In (4), γ_i and γ_0 are, respectively, the propagation constants of the samples $i = 1, 2$ and air-filled waveguide. $\lambda_0 = c/f$ and $\lambda_c = c/f_c$ correspond to the free-space and cut-off wavelengths; and f, f_c, and c are the operating and cut-off frequencies and the speed of light in vacuum respectively; and $\varepsilon_{ri}^* = \varepsilon_{ri}' - j\varepsilon_{ri}''$ are the complex relative permittivities of the standard dielectric ($i = 1$) and the MUT ($i = 2$) samples.

To remove the influence of the unknown matrices T_X and T_Y, we do the following:

$$M_1 \cdot M_2^{-1} = T_X \cdot T_{ref1} \cdot T_1 \cdot T_{ref1}^{-1} \cdot T_{ref2} \cdot T_2^{-1} \cdot T_{ref2}^{-1} \cdot T_X^{-1} \tag{6}$$

It is clear that the effect of T_Y is eliminated. View to further simplifying (6) and eliminating the need of another dielectric sample which the permittivity must be well known, we take air ($\varepsilon_{r1}^* = 1, \gamma_1 = \gamma_0, and \Gamma 1 = 0$) as a standard dielectric; which is equivalent to an empty waveguide measurement. T_{ref1} is then equal to the unit square matrix. And it is obvious from (6) that $M_1 M_2^{-1}$ and $T_1 \cdot T_{ref2} \cdot T_2^{-1} T_{ref2}^{-1}$ are similar matrices, this implies that they have

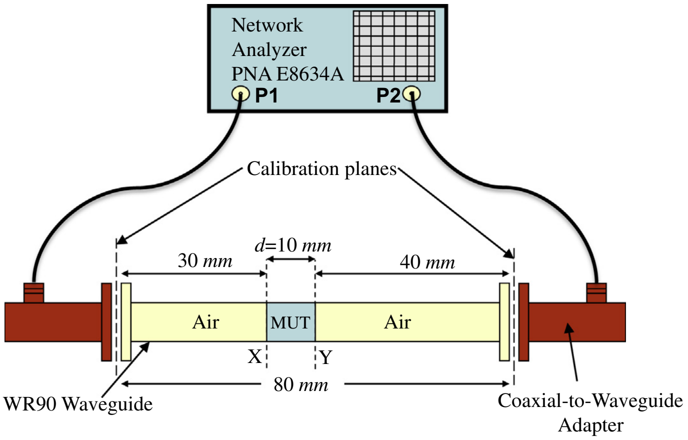

Fig. 1. The waveguide measurement setup for complex permittivity determination by the proposed Transmission/Reflection method. The material under test MUT of thickness $d = 10$ mm is filled inside the rectangular waveguide WR90 which is connect to the Vector Network Analyzer via the coaxial-to-waveguide adapter.