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Trend extraction based on separations of consecutive empirical mode decomposition components in Hilbert marginal spectrum

Zhijing Yang [a,b], Bingo Wing-Kuen Ling [c,*], Chris Bingham [b]

[a] Faculty of Computer, Guangdong University of Technology, Guangzhou 510006, China
[b] School of Engineering, University of Lincoln, Lincoln, Lincolnshire LN6 7TS, United Kingdom
[c] Faculty of Information Engineering, Guangdong University of Technology, Guangzhou 510006, China

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ABSTRACT

Extracting the underlying trends is an important tool for the analysis of signals. This paper presents a novel methodology for extracting the underlying trends of signals based on the separations of consecutive empirical mode decomposition (EMD) components in the Hilbert marginal spectrum. A signal is initially represented as a sum of intrinsic mode functions (IMFs) obtained via the EMD. The Hilbert marginal spectrum of each IMF is then calculated. The separations of two consecutive IMFs in the Hilbert marginal spectrum are estimated based on their correlation coefficients. The group of the last several IMFs in which the IMFs are close to each other in the Hilbert marginal spectrum will be used for the representation of the underlying trend of the signal. Extensive experimental results are presented to illustrate the rationale and the effectiveness of the proposed method.

1. Introduction

Time series often consists of global changes and irregularities. Typically, the underlying trend of a signal refers to these global changes [1]. Extracting the underlying trends plays an important role in the analysis of signals from industrial turbines [2], power systems [3], plant industries [4], economics [5] and biomedical sciences [6,7].

Existing methods for extracting the underlying trends of signals can be broadly divided into three distinct categories viz. model based approaches [8], signal processing methods [9,35,36] and nonparametric trend prediction methods [10]. Among them, the signal processing methods are the most promising approach.

Lowpass filtering is the oldest method [23] for extracting the underlying trends of signals. However, lowpass filtering is a linear time invariant method. The averaging effect introduced by the weighted sum in the convolution operator will result to the failure of tracking sudden jumps in the signals. This effect is particular serious if the filter lengths are long. For filters with short lengths such as the filters with the impulse responses only consisting of three points (the previous point, the current point and the future point), they have extremely poor frequency responses [23,25–34]. That means, both the passband ripples and the stopband ripples are very large. Hence, the filters cannot get rid of the noise and allow irregularities to pass through. Also, the filters will introduce the delays to the signals and there are boundary effects. Although the delays introduced by the lowpass filtering can be compensated easily as well as the passbands and the stopbands of the filters can be changed, the changes on the passbands and the stopbands are signal dependent in which the required adaptive changing rules are too complicated to be implemented in practical situations.

Recently, it is found that the underlying trends of some signals can be extracted by only retaining the last intrinsic mode function (IMF) obtained by the empirical mode decomposition (EMD) which is a powerful tool for analyzing nonlinear and non-stationary signals [12–22]. Since the last IMF is always monotonic, this approach fails to extract

* Corresponding author. Tel.: +86 20 3932 2246; fax: +86 20 3932 2252.
E-mail address: yongquanling@gdut.edu.cn (B.Wing-Kuen Ling).

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the underlying trends of many signals [13] such as those with the underlying trends are not monotonic. In general, more than one IMF should be retained. However, there does not exist any method for the analysis of the EMD and for the adaptive selection of these IMFs. This problem still remains unsolved and it is challenging.

This paper proposes a novel methodology for extracting the underlying trends based on the separation of the EMD components represented in the Hilbert marginal spectrum. The outline of this paper is as follows. Section 2 briefly reviews the algorithm of EMD and the definition of the Hilbert marginal spectrum. In Section 3, a novel methodology for extracting the underlying trends based on the separation of the EMD components represented in the Hilbert marginal spectrum is proposed. Section 4 presents experimental results. Finally, a conclusion is drawn in Section 5.

2. Review on the EMD of signals and the definition of the Hilbert marginal spectrum

2.1. EMD

The EMD of signals is based on the direct detection of the envelops of the signals. Since this decomposition is based on the local time scale characteristic of the signals, it is applicable to nonlinear and nonstationary processes. In principle, a signal is represented via the sum of IMFs, from which the instantaneous frequencies can be analyzed using the Hilbert transform. The definition of the IMF is as follows [11]:

Definition 1. IMF A function is considered as an IMF if it satisfies the following two conditions: (1) the number of extrema and the number of zero crossing points are equal, or their difference is no more than 1; and (2) its local mean is zero.

Given a signal $x(t)$, where $t \in R$, the principle of the EMD of $x(t)$ can be interpreted as follows.

(1) Initialization: $r_0(t) = x(t)$ and $i = 1$.
(2) Compute the ith IMF $c_i(t)$ using the following iterative procedure:
  (a) Let $d_0(t) = r_{i-1}(t)$ and $j = 1$;
  (b) Identify all the local maxima and minima of $d_{j-1}(t)$;
  (c) Generate the upper and lower envelopes of $d_{j-1}(t)$ using the cubic spline interpolation (typically), denoted as $e_{up}(t)$ and $e_{low}(t)$;
  (d) Calculate the local mean by $m(t) = (e_{up}(t) + e_{low}(t))/2$;
  (e) Sifting: $d_j(t) = d_{j-1}(t) - m(t)$;
  (f) If $SD = \int_t \frac{|m(t)|^2}{|d_{j-1}(t)|^2} dt < \kappa$ (κ is usually selected as 0.3), then let $c_i(t) = d_j(t)$ and go to step (3); otherwise, increment the value of j and go back to step (b).
(3) Let $r_i(t) = r_{i-1}(t) - c_i(t)$.
(4) If $r_i(t)$ satisfies the properties of IMF or it is a monotonic function, then the decomposition is complete; otherwise, increment the value of i and go back to step (2).

When the decomposition is complete, the original signal can be represented as:

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t). \tag{1}$$

Here, consider the final $r_n(t)$ as an IMF denoted as $c_{n+1}(t)$, so we have

$$x(t) = \sum_{i=1}^{n+1} c_i(t). \tag{2}$$

The EMD of a signal results to a finite set of IMFs with each based on a distinct time scale. Since the decomposition is based on the nature of the signals, it is an adaptive signal decomposition algorithm. The first component has the smallest time scale and it typically corresponds to the noise parts of the signal. As the decomposition proceeds, the time scale increases. The last component has the largest time scales. It typically corresponds to the underlying trend of the signal.

2.2. Hilbert marginal spectrum

Having decomposed a signal into a finite number of IMFs, the instantaneous frequencies of the signal can be calculated via the Hilbert transform [11]. The Hilbert transform is initially applied to each IMF $c_i(t)$ as follows:

$$z_i(t) = c_i(t) + j\tilde{H}[c_i(t)] = a_i(t)e^{j\theta_i(t)}, \tag{3}$$

where the Hilbert transform is defined as

$$\tilde{H}[c_i(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c_i(t')}{t - t'} dt', \tag{4}$$

and P indicates the Cauchy principal value. Clearly,

$$a_i(t) = \sqrt{(c_i(t))^2 + (\tilde{H}[c_i(t)])^2} \quad \text{and} \quad \theta_i(t) = \arctan\left(\frac{\tilde{H}[c_i(t)]}{c_i(t)}\right), \tag{5}$$

are the instantaneous amplitude and phase of $c_i(t)$, respectively. Hence, $x(t)$ can then be expressed as

$$x(t) = \mathrm{Re}\left(\sum_{i=1}^{n+1} a_i(t)e^{j\theta_i(t)}\right). \tag{6}$$

From (5), the instantaneous frequency of $c_i(t)$ is denoted as $\omega_i(t)$ and it is defined as

$$\omega_i(t) = \frac{d\theta_i(t)}{dt}. \tag{7}$$

From (6), it can be seen that the IMF representation provides a generalized form of the representation of signals such includes the Fourier series representation, but circumvents the restriction of constant amplitude and fixed harmonic frequency of the Fourier series representation.