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Technical note

On the thermal characterization of solids by the relaxation photothermal technique

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ABSTRACT

The photothermal relaxation method is widely used for determining the volume thermal capacity of solids. The complete mathematical treatment necessary for obtaining the analytical solution of the heat diffusion equation considering the experimental setup usually used is presented. The conditions of applicability of the method are discussed.

1. Introduction

The study of thermal properties of solids such as thermal conductivity (k), diffusivity (α) and volume heat capacity (C_v) is of interest in different applications which require the use of materials at elevated temperatures. At the same time the performance of certain devices such as thermoelectric generator and cooling are closely related with the thermal properties of materials used in the device. The physical meaning of the thermal conductivity is the rate of the heat transfer by conduction in steady-state phenomena while for the thermal diffusivity it is the rate of heat transfer by conduction in time-dependent temperature phenomena. The volume heat capacity determines the amount of heat required to change the temperature of the unity of volume of substance by 1 K.

The thermal conductivity can be measured by the 3-ω method and the transient plane source technique [1] and [2] while the heat capacity is usually determined by calorimetric methods [3]. The most popular method for measuring the thermal diffusivity of solids is the Flash laser method [4] and Photoacoustic method [5]. This last method has been also proposed for volume heat capacity measurement [6].

Actually the photothermal relaxation methods are intensively used for thermal characterization of solids. Their principle of work is based on the measurement of the time variation of the temperature in an adiabatically isolated sample after disturbing its state of equilibrium by light irradiation. A heat is developed in the sample as a consequence of the non-radiative de-excitation process following the light absorption. The thermal relaxation process is characterized by a relaxation time which is closely related with thermal properties of solid. These methods are very attractive because they offer the advantage of an inexpensive and simple experimental setup as well as they are low-time consumer, non-destructive and they allow measuring a small size sample. The Flash laser technique is the most popular of them. In this technique the sample is irradiate with a light pulse of several hundreds of microseconds. A photo-calorimetric relaxation method for determining the heat capacity of solids has been also proposed [7]. Other approach was proposed in [8] by using a similar setup that Flash laser technique but the time-varying temperature was measured after exposing the sample under continuous white-light illumination. In order of obtaining a practical mathematical expression for the time dependence of temperature, the authors resolved the one-dimensional heat transfer equation considering the radiation losses. Details of the mathematical procedure used were

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not shown. The method allows obtaining the heat capacity but the thermal conductivity and thermal diffusivity cannot been obtained independently. Recently the thermal gradient in the sample when performing the same measurement was studied by analyzing the analytical solution of the heat diffusion equation [9]. It was enough for the goal of these papers to present the partial solution of the heat diffusion equation. The case of thermodynamic equilibrium limit for infinitely long times was studied. A similar analysis was reported in [10]. This work aims to show the complete analytical solution of heat diffusion equation considering the experimental setup proposed by Mansanares et al. [8]. The conditions of applicability of the formula (11) of this paper are discussed.

2. Theoretical details

When the front surface of a parallel plate sample is uniformly heated by a continuous light illumination, the analytical spatial and temporal variation of the temperature in the sample can be obtained by solving the heat diffusion equation. If the thickness of the homogeneous sample is lesser than the dimensions of the sample area, the heat absorbed at the front face diffuses one-dimensionally to the rear surface and thus we can work with the one-dimensional heat diffusion equation. The temperature variation can be monitored on the rear surface by a temperature sensor.

Let us consider that the sample thickness is (L), the initial sample temperature is (T_o), the incident light power by unit of surface for $t > 0$ is (P_o) and the heat diffuses in the x-direction of a Cartesian system of coordinates. For this case the one-dimensional heat diffusion equation can be written in this way:

$$\frac{\partial v_{(x,t)}}{\partial t} - \alpha \frac{\partial^2 v_{(x,t)}}{\partial x^2} = 0 \quad \text{for } 0 < x < L, \;\; t > 0 \tag{1}$$

being α the thermal diffusivity and

$$v_{(x,t)} = T_{(x,t)} - T_o \tag{2}$$

The initial and boundary conditions are as it follows:

$$v|_{t=0} = 0 \tag{3}$$

$$H_1 v_{(0,t)} - k \left.\frac{\partial v_{(x,t)}}{\partial x}\right|_{x=0} = P_0 \quad \text{for x} = 0 \tag{4}$$

$$H_2 v_{(L,t)} + k \left.\frac{\partial v_{(x,t)}}{\partial x}\right|_{x=L} = 0 \quad \text{for x} = L \tag{5}$$

being k the thermal conductivity.

These boundary conditions take into account the radiation and convection losses by means of overall heat transfer coefficient H. The subscripts 1 and 2 in H are related with the front and rear surface respectively. H takes the form [annex in 9],

$$H = h_{\text{conv}} + 4\sigma\varepsilon T_o^3 \tag{6}$$

where h_{conv} is the convective heat transfer coefficient, ε is the total emissivity of the surface and α is the Stefan–Boltzmann constant.

It should be observed that in the paper [9] there are misprints in formulas (1) and (5).

The heat equation is a homogenous parabolic partial differential equation with non-homogeneous boundary equations. The solution of this equation can be found in this way

$$V_{(x,t)} = \varepsilon_{(x)} + u_{(x,t)}$$

where is the solution of the differential equation $\frac{d^2\omega}{dx^2} = 0$ with the same boundary conditions (4) and (5) but substituting v by ω.

$u_{(x,t)}$ is the solution of the differential equation

$$\frac{\partial u_{(x,t)}}{\partial t} - \alpha \frac{\partial^2 u_{(x,t)}}{\partial x^2} = 0 \tag{7}$$

with the boundary conditions (4) and (5) completely homogeneous (Robin conditions) and substituting v by u. Evidently the initial condition for $u_{(x,t)}$ is

$$u|_{t=0} = -\omega_{(x)} \tag{8}$$

Solving the equation for ω we obtain,

$$\omega_{(x,t)} = \frac{-H_2 P_o x + P_o(H_2 L + k)}{(H_1 + H_2)k + H_1 H_2 L} \tag{9}$$

A similar problem to that of $u_{(x,t)}$ can be found in [11]. The solution takes the form

$$u_{(x,t)} = \sum_{n=1}^{\infty} A_n \chi_{n(x)} e^{-\alpha\gamma_n^2 t} \tag{10}$$

where χ_n will be written conveniently as it follows

$$\chi_{n(x)} = \cos\gamma_n x + \frac{H_1}{k\gamma_n}\sin\gamma_n x \tag{11}$$

and γ is the solution of the transcendental equation

$$\tan\gamma_n L = \frac{\frac{1}{k} L(H_1 + H_2)\gamma_n L}{\gamma_n^2 L^2 - \frac{H_1 H_2 L^2}{k^2}} \tag{12}$$

Considering that χ_n is an orthogonal function for determining A_n we can use the similar procedure used usually for the coefficients of a Fourier series transforming the expression (10) by using condition (8). Thus A_n can be calculated in this way,

$$A_n \int_0^L \chi_{n(x)}^2 dx = -\int_0^L \omega_{(x)} \chi_{n(x)} dx \tag{13}$$

It should be observed that there is a misprint in the denominator of the transcendental Eq. (16) and in the norm of function χ_n in the formula (17) of [9].

The final expression for the variation of the temperature in the sample ($v_{(x,t)} = \Delta T_{(x,t)}$) when the front surface is heated with a continuous light irradiation is,

$$\begin{aligned}\Delta T_{(x,t)} = {} & \frac{-H_2 P_o x + P_o(H_2 L + k)}{(H_1 + H_2)k + H_1 H_2 L} \\ & - 2\sum_{n=1}^{\infty} \frac{\gamma_n^2}{\left(\gamma_n^2 + \frac{H_1^2}{k^2}\right)L + 2\frac{H_1}{k}} \left(\cos\gamma_n x + \frac{H_1}{k\gamma_n}\sin\gamma_n x\right) e^{-\alpha\gamma_n^2 t} \\ & \times \int_0^L \omega_{(x)} \chi_n dx\end{aligned} \tag{14}$$