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Time-domain filtering for estimation of linear systems with colored noises using recent finite measurements

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ABSTRACT

To date, finite impulse response (FIR) filters have been proposed to estimate linear systems with white Gaussian noises, but to the best of our knowledge, no solution exists for linear systems with colored noises. In this paper, we propose a new FIR filter to estimate linear state-space models with both process and measurement noises through state augmentation. In addition, we suggest a modified form of the colored-noise FIR filter to deal with the computational burden and singularity problem. Numerical examples are presented to describe the effectiveness of the colored-noise FIR filter.

1. Introduction

The Kalman filter [1] is a mathematical tool that provides an effective computational method to estimate the state of systems which minimizes the mean-squared estimation error. The Kalman filter is known to exhibit poor performance as well as the divergence phenomenon when temporary uncertainties are applied to the systems. This is because the Kalman filter with its infinite impulse response (IIR) structure utilizes all past observations accomplished by equal weighting and tends to become accumulated during its implementation [2–4]. As an alternative to the Kalman filter, finite impulse response (FIR) filters [5–14] have been proposed in various engineering problems such as target tracking, wireless positioning systems, and control system design. FIR filters utilize only finite measurements and inputs in the most recent time interval $[k-N;k]$. Given their structural characteristics, FIR filters have bounded input-bounded output (BIBO) stability and robustness to temporary model uncertainties and round-off errors.

Despite these advantages of FIR filters, we cannot design FIR filters for colored-noise systems, because they are applicable to only white-Gaussian-noise systems. In real physical systems, the assumption of white noise is invalid because of multipath and slow signal propagation. In many cases, process and measurement noises tend to be colored. Thus, it is desirable to design a new FIR filter for colored-noise systems. However, to the best of our knowledge, the FIR filtering problem for colored-noise systems has never been solved. Thus, it remains unresolved and challenging.

In this paper, for the first time, we propose a new FIR filter for linear state-space models with colored process and measurement noises. In contrast to existing works on FIR filtering, this work deals with systems with colored noises. State augmentation is applied to the design of the a new FIR filter for systems with colored noises. In order to deal with the computational burden and singularity problem in colored-noise FIR filtering, we present a modified form of the proposed colored-noise FIR filter. Through numerical simulations on the F-404 engine model, we verify the effectiveness of the proposed approach by comparison with the Kalman filtering approach for colored-noise systems [15].

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This paper is organized as follows. In Section 2, a new solution to the colored-noise FIR filtering problem and its modified form are proposed for discrete-time state-space models. In Section 3, numerical examples are presented. Finally, conclusions are stated in Section 4.

2. Main results

2.1. FIR filter design for linear systems with colored noises

In this subsection, we suppose that colored process and measurement noises are both static and that they can be obtained by passing a Gaussian white noise in a dynamic system. Consider the state-space model

$$\begin{aligned} x_k &= Ax_{k-1} + Gw_{k-1}, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where $x_k \in \Re^n$ is the state, $y_k \in \Re^q$ is the output, $w_k \in \Re^p$ is the process noise with a covariance matrix Q_k, and $v_k \in \Re^l$ is the measurement noise with a covariance matrix R_k. w_k and v_k are colored noises that have autocorrelations unlike white noise. In general, a colored noise can be generated by filtering white noise. We suppose that w_k and v_k are modeled by the following dynamic systems:

$$\begin{aligned} w_k &= \Psi w_{k-1} + \zeta_{k-1}, \\ v_k &= \Phi v_{k-1} + \rho_{k-1}, \\ \zeta_k &\sim N(0, Q_\zeta), \\ \rho_k &\sim N(0, Q_\rho), \end{aligned} \tag{2}$$

where $\Psi \in \Re^{p\times p}$ and $\Phi \in \Re^{l\times l}$ are state transition matrices. ζ_{k-1} is a white noise with zero-mean, which is not correlated with w_k. Likewise, ρ_{k-1} is a white noise with zero-mean, which is also not correlated with v_k. Thus, we can consider that covariances Q_k and R_k correspond to

$$E(w_k w_{k-1}^T) = E(\Psi w_{k-1} w_{k-1}^T + \zeta_{k-1} w_{k-1}^T) = \Psi Q_{k-1} + 0, \tag{3}$$

$$E(v_k v_{k-1}^T) = E(\Phi v_{k-1} v_{k-1}^T + \rho_{k-1} v_{k-1}^T) = \Phi R_{k-1} + 0, \tag{4}$$

respectively. We can see that w_{k-1} and v_{k-1} are independent of ζ_{k-1} and ρ_{k-1}, respectively. In (3) and (4), we observe that w_{k-1} and v_{k-1} are colored noises. To solve the discrete-time FIR filtering problem with colored process and measurement noises, we introduce system augmentation as follows:

$$\begin{bmatrix} x_k \\ w_k \\ v_k \end{bmatrix} = \begin{bmatrix} A & G & 0 \\ 0 & \Psi & 0 \\ 0 & 0 & \Phi \end{bmatrix} \begin{bmatrix} x_{k-1} \\ w_{k-1} \\ v_{k-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} I_{k-1} \\ \zeta_{k-1} \\ \rho_{k-1} \end{bmatrix}, \tag{5}$$

$$y_k = [\,C \quad 0 \quad I\,] \begin{bmatrix} x_k \\ w_k \\ v_k \end{bmatrix} + 0.$$

These equations can be represented as

$$\begin{aligned} \bar{x}_k &= \overline{A}\bar{x}_{k-1} + \overline{G}\bar{w}_{k-1}, \\ y_k &= \overline{C}\bar{x}_k + \bar{v}_k, \end{aligned} \tag{6}$$

where

$$\bar{x}_k \triangleq \begin{bmatrix} x_k \\ w_k \\ v_k \end{bmatrix}, \quad \bar{w}_{k-1} \triangleq \begin{bmatrix} I_{k-1} \\ \zeta_{k-1} \\ \rho_{k-1} \end{bmatrix}, \quad \bar{v}_{k-1} \triangleq 0, \tag{7}$$
$$\overline{A} \triangleq \begin{bmatrix} A & G & 0 \\ 0 & \Psi & 0 \\ 0 & 0 & \Phi \end{bmatrix}, \quad \overline{C} \triangleq [\,C \quad 0 \quad I\,], \quad \overline{G} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

We obtain a new augmented state-space model having a new state $\bar{x}$, state transition matrix $\overline{A}$, measurement matrix $\overline{C}$, process noise $\bar{w}$, and measurement noise $\bar{v}$. The new noise covariances are

$$\begin{aligned} E[\bar{w}_k \bar{w}_k^T] &= \left[\begin{pmatrix} I_k \\ \zeta_k \\ \rho_k \end{pmatrix} \begin{pmatrix} I_k^T & \zeta_k^T & \rho_k^T \end{pmatrix} \right] \\ &= \begin{bmatrix} I & 0 & 0 \\ 0 & Q_\zeta & 0 \\ 0 & 0 & Q_\rho \end{bmatrix} = \overline{Q}, \end{aligned} \tag{8}$$

$$E[\bar{v}_k \bar{v}_k^T] = 0 = \overline{R}, \tag{9}$$

On the most recent time horizon $[k-N, k]$, the FIR filter for the new augmented system (6) is represented as

$$\hat{\bar{x}}_{k|k-1} = \sum_{i=k-N}^{k-1} H_{k-i} y_i = HY_{k-1}, \tag{10}$$

where H is the gain matrix and Y_{k-1} is the finite number of measurements, which are defined as

$$\begin{aligned} H &\triangleq [H_N \quad H_{N-1} \cdots H_1], \\ Y_{k-1} &\triangleq \left[y_{k-N}^T \quad y_{k-N+1}^T \quad \cdots \quad y_{k-1}^T\right]^T, \end{aligned} \tag{11}$$

respectively. Y_{k-1} can be represented in terms of state and noises as

$$Y_{k-1} = \tilde{\tilde{C}}_N x_{k-N} + \tilde{\tilde{G}}_N \overline{W}_{k-1} + \overline{V}_{k-1}, \tag{12}$$

where

$$\begin{aligned} \overline{W}_{k-1} &\triangleq \left[\bar{w}_{k-N}^T \ \bar{w}_{k-N+1}^T \ \cdots \ \bar{w}_{k-1}^T\right]^T, \\ \overline{V}_{k-1} &\triangleq \left[\bar{v}_{k-N}^T \ \bar{v}_{k-N+1}^T \ \cdots \ \bar{v}_{k-1}^T\right]^T, \end{aligned} \tag{13}$$

$\tilde{\tilde{C}}_N, \tilde{\tilde{G}}_N, \tilde{\tilde{Q}}_N$, and $\tilde{\tilde{R}}_N$ are obtained as

$$\tilde{\tilde{C}}_N \triangleq \begin{bmatrix} \overline{C} \\ \overline{CA} \\ \overline{CA}^2 \\ \vdots \\ \overline{CA}^{N-1} \end{bmatrix}, \quad \tilde{\tilde{G}}_N \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \overline{CG} & 0 & \cdots & 0 & 0 \\ \overline{CAG} & \overline{CG} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{CA}^{N-2}\overline{G} & \overline{CA}^{N-3}\overline{G} & \cdots & \overline{CG} & 0 \end{bmatrix}, \tag{14}$$

$$\tilde{\tilde{Q}}_N \triangleq [\mathrm{diag}(\overbrace{\overline{Q}, \overline{Q}, \cdots, \overline{Q}}^{N})],$$

$$\tilde{\tilde{R}}_N \triangleq [\mathrm{diag}(\overbrace{\overline{R}, \overline{R}, \cdots, \overline{R}}^{N})].$$

where $\overline{C}, \overline{G}, \overline{Q}$, and $\overline{R}$ are obtained by system augmentation. By using Lemma 4.3 in [16], the gain matrix H of the colored-noise FIR filter can be obtained by the following correspondences: