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Mathematical modelling in the analysis and design of hard and soft measurement systems ☆

Sanowar Khan *, Ludwik Finkelstein

City University London, Instrumentation and Sensors Research Centre, School of Engineering and Mathematical Sciences, Northampton Square, London EC1V 0HB, UK

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ABSTRACT

This paper briefly looks into the role and extent of mathematical modelling in the design and analysis of measurement systems, especially measurement sub-systems in the form of instruments and instrument elements. These fall within the classical domain of hard measurement (physical, strongly-defined measurement). It also examines the role and use of mathematical modelling in the area of soft measurement (non-physical, weakly defined measurement). These constitute two sub-sets of widely-defined measurement. Based on a number of examples it demonstrates the use of modern modelling techniques in the design and analysis of sub-systems in measurement technology. In doing so, it focuses on the scope and importance of physical modelling at a sub-system level, which ultimately contributes to modelling activities at a global systems level.

1. Introduction

With the ever increasing availability of computing technology significant progress is being made in the application of mathematical modelling techniques, especially numerical techniques for modelling, CAD, performance prediction and validation of measurement systems and sub-systems. Mathematical modelling is a key enabling tool and a means by which the functioning of systems and sub-systems can be predicted from a description of its physical principles, geometric features and material properties.

It is now widely understood that a model of a system is the description of the system in a formal language, such that relations between symbols in statements in the language imply and are implied by relations between the objects and attributes of the system and its components [1]. In other words, a model can be looked upon as the representation of a physical process and possesses the essential attributes of that physical process. Models are extensively used in design and by modelling it is meant the study of the mechanisms inside a system, and through using basic laws and relationships, a model is inferred. In terms of representation schemes, there could be linguistic, pictorial and mathematical models [2,3]. This paper primarily focuses on mathematical models in which physical sub-systems are described as a set of mathematical relations (e.g. equations, discrete data, etc.) representing the physical processes, properties and behaviour of the sub-systems. At the same time it also briefly investigates current developments and challenges in mathematical modelling and analysis in the domain of soft measurement systems.

2. Mathematical models of instruments as measurement sub-systems

In modern instrumentation, information is generally carried by electrical signals. The analysis and design of these signals is generally performed by standard methods

☆ I presented a shorter version of this paper, our last joint paper at the IMEKO Joint TC1 + TC7 + TC13 Symposium, Jena, Germany, 31 August-2 September, 2011 almost at the same time as Ludwik's funeral took place in London.

* Corresponding author.
E-mail address: S.H.khan@city.ac.uk (S. Khan).

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of signal theory. The signal and information processing components of modern instrumentation are generally standard components and are described by functional models. They are commonly implemented by standard information technology hardware and software and are analysed and designed by the general methods of information technology. However, the sensors and actuators of instrument systems are required to be analysed and designed in terms of their physical embodiment and function. Their analysis and design thus require special methods.

The ultimate objective of developing mathematical models and computer-aided methods of design of instrument sub-systems is the development of integrated computer environments in which the total design of these systems can be undertaken. Such an environment would be based on a modern model of the design process, based on the concepts of knowledge processing and problem solving. A model of the design process, based on a blackboard architecture, has been proposed and discussed in [4]. There continue to be developments reported in the literature of knowledge engineering, and artificial intelligence, problem solving and design. Models of the design process based on these advances and other modelling approaches [5] have significant conceptual value for measurement science and education in the field. However, there is considerable gap between these models and practical application. With the availability of state of the art numerical modelling tools, both generic and application-specific, in all areas of measurement science and technology this gap between the reality and its mathematical representation is rapidly shrinking. However, significant challenges are still being posed by large and complex systems in both hard (e.g. biological [6]) and soft measurement (e.g. economic, financial, etc. [7]) areas.

3. Evolution of mathematical modelling of instruments as measurement sub-systems

Design by computer modelling and simulation of measurement sub-systems is based on their appropriate representation by models, which can be handled by computers. The general concepts of instrument modelling have been considered in [2,3]. For example, in the case of sensors and actuators two kinds of models are used: power flow models which represent the functional relationship between physical inputs and physical outputs, and embodiment models that represent these relationships in terms of the geometry and material properties of the embodiment. Power flow models have seen substantial application. They are extensively used in the modelling and design of systems that consist of interacting components with diverse forms of energy. Mechatronics is an area in which such models are extensively and effectively used. In general instrumentation they provide a means of representing archetype models of sensors and actuators [2]. They also are tools for modelling the interaction of sensors and the system being sensed, and that of actuators with the system upon which they act.

The main advances in these types of models have been in the development and application of computer software that automates model formulation and solution of system models. Significant advances have been made in languages and computer packages for power flow models. In particular they are bond graphs [8–12], Modelica [13–15], Scilab/Scicos [16], MATRIXx/SystemBuild and the widely applied MATLAB [17–20]. The main requirement in the modelling of measurement sub-systems is for embodiment models. It is in this area where the principal advances have been made for sensors and actuators.

Qualitative, computer implemented, models have significant application potential in the description and in the design concept generation of complex instrumentation systems. Such models are making progress and they are beginning to provide useful insight and find effective practical applications [21,22].

4. Mathematical modelling in soft measurement systems

Soft measurement (or weakly-defined measurement) is a sub-set of widely-defined measurement which is not strongly defined. It constitutes representation by symbols of properties of entities of the real world, based on an objective empirical process, but lacks some, or all, of the distinctive characteristics of strongly-defined measurement [23–27]. It lacks well-formed theories and involves predominantly non-physical sciences. Examples might be: psychology [28–30] – intelligence [31], attitude, subjective perception of physical stimuli (colour, odour); sociology [32,33] – class, status, segregation, poverty; economics, linguistics – measurement of phonological, lexical, grammatical and other attributes of natural language communication. In general, a soft system is any system for which there is not an adequately complete, empirically validated theory. This embraces much of the psychological, social and economic domains [7]. Modelling and analysis by modelling of such soft systems pose significant challenges and it is made much more complicated by the fact that these systems involve human action, perception [34], feeling, decisions and the like. They can thus not be described by a system of invariant relations [24,25,27].

The main difficulties for modelling in soft measurement systems stem from the fact that soft measurement systems (a) are based on ill-defined concept of quality, (b) have significant uncertainty in the empirical relational system that it represents, (c) have symbolic relational system with limited relations defined on it, and (d) have no adequate theory relating the measurement to other measurements in the same domain.

In fact the whole area of widely-defined measurement, which is needed for the wide and diverse application of measurement, offers significant conceptual problems, compared with measurement in the physical sciences. These are in relation to: (i) experiments and observation – economic and biological systems in which it is not always possible for models to represent reality adequately, (ii) replicability – in relations with measurands: psychological and social sciences – humans, complex systems where it may not be possible to formulate adequate models to represent such systems, (iii) utility – value