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Specific accelerating factor: One more tool in motor sizing projects

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A B S T R A C T

This work is focused on improvement of motor sizing methods based on the *accelerating factor* [4]. This quantity is used to seek commercial motor–gearbox couples for mechatronics; unfortunately, it does not allow comparison of different acceptable solutions. The designer must therefore make a choice based on experience, instinct and poor data of catalogues. A per-unit-length value of the accelerating factor (*specific accelerating factor*) gives instead a rough but consistent idea of a servo-motor quality, for a more systematic and thoughtful selection. The specific accelerating factor represents a "benchmark" for a motor, it's easy to calculate from datasheets and was defined using well known notions of electromechanical design (under certain assumptions).

1. Introduction

A classical electrotechnical approach in motor design is finding precise load performance requirements in order to built the most appropriate customized machine for the application. This is the practice for big motors and/or for particularly critical applications, in which development costs are justified. In automation or mechatronic field, instead, developing brand new motors for every different application is basically unsustainable; despite that, high performances are equally requested. Only solution is choosing as best as possible the motor from a commercial list of existing ones. This procedure is called *motor sizing*, and it's absolutely different from the previous approach; electrical machines are seen as "black-boxes", and manufacturer's catalogues are generally the only available data.

The criteria for the correct choice of electric brushless motor and gearbox in the automation field have been widely studied since 1980s [1]. Some procedures consider purely inertial loads applied to the motor [2], while others consider a more generic load [3,4]. Generally, as presented in [5], the mechanical efficiency and the inertia of the transmission are not considered until the verification phase, while in [6] such effects are taken into account since the beginning of the machine design.

Whatever method is used, two situations can happen:

– More than one couple motor–gearbox is suitable for the given application;
– None known motor–gearbox is suitable for the given application;

Considering this, a good sizing method should also answer the following designer's questions:

(a) *"For the many suitable motor–gearboxes I've found, can I calculate, with poor datasheets information, a parameter that makes a performance classification for the given application?"*
(b) *"Can I find an index, theoretic or practical, that gives the idea how a certain motor is far from top quality performances?"*
(c) *"I haven't found a suitable motor in my database, but can I hope to find something good in another database/catalogue or my load requirements are too heavy?"*

Methodologies presented in literature [1–8] are primarily based on the thermal analysis, searching for the maximum torque the motor can exert without overheating. Most of these methods define a parameter starting from the characteristics of the motor, independent by the given application; this quantity can be calculated using only the information collected in the manufacturer catalogs. Several different definitions of this parameter are present in the literature. One of the most common is based on the relationship between the motor rated torque and its moment of inertia.

This parameter is called "accelerating factor" or "continuous duty power rate factor" or simply "power rate". The power rate is a well known concept in literature and different considerations have been derived to handle it as a usefulness motor coefficient in comparison with the torque-to-inertia ratio [9–11].

The accelerating factor is useful for assessing whether a motor is able to perform a given task, but it's not sufficient to answer previous questions; no comparison between suitable motors is available (two brushless motors, very different in terms of construction, size and weight, may have the same accelerating factor), and a superior limit of the accelerating factor does not exist. For

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this reason the designer of an automatic machine has difficulties in understanding whether the choice done is the best or not.

This work is focused on the analysis of the accelerating factor from a phenomenological point of view, analyzing data available on catalogs of motor manufacturers and trying to find a relationship between this parameter and the construction features of a brushless motor. The purpose is not to find a formula to help the motor manufacturer to improve the performance of their devices, but to help the designer to choose the most valid motor for his application. In particular, through the results of this activity, the designer has a tool to assess the chosen motor and to understand if something better could be available on the market; a benchmark for the accelerating factor would help in selecting the best motor–transmission coupling.

The writ is structured as follows: in the first section a brief compendium of [4] reprises the accelerating factor sizing method. In the second section the performance of some brushless motors are compared. In the third section the investigation on the electromechanical model of a brushless motor is discussed and in the fourth section a new parameter useful to compare the performance of different motors is introduced. Finally conclusions are drawn in the last section.

Symbols used in the paper are in Table 1.

2. Selection criterion

2.1. The model

A complex automatic machine can be divided into simpler subsystems, able to operate one degree of freedom. As shown in Fig. 1 they can be summed up in three key parts: servo-motor, transmission and load. While the load characteristics are completely known as they depend on the machine task, the motor and the transmission are unknown until their selection.

Brushless motors working range consists of a continuous working zone (rated torque zone) and a dynamic zone (related to the maximum motor torque $T_{M,max}$) (Fig. 2).

Knowing only essential data from catalogs, the rated torque is usually considered constant and equal to $T_{M,N}$ up to maximum motor speed $\omega_{M,max}$ [12].

Frequently, in industrial applications, the machine task is periodic with cycle time t_a much smaller than the motor thermal time constant. The motor behavior can therefore be analyzed through the root mean square (rms) value of T_M defined as:

$$T_{M,rms} = \sqrt{\frac{1}{t_a}\int_0^{t_a} T_M^2 \, dt} \tag{1}$$

namely the torque that, acting steadily over the cycle, generates the total energy dissipation.

The selection of the actuator requires to check the following conditions:

- rated motor torque:

$$T_{M,rms} \leqslant T_{M,N}; \tag{2}$$

- maximum motor speed:

$$\omega_M \leqslant \omega_{M,max}; \tag{3}$$

- maximum motor torque:

$$T_M(\omega_M) \leqslant T_{M,max}(\omega_M). \tag{4}$$

Fig. 2 graphically show the meaning of inequalities (2)–(4).

Conditions (2)–(4) are well known in literature and represent the starting point of all the procedures for motor and reducer selection.

2.2. The accelerating factor and the load factor

The motor torque T_M can be written as:

$$T_M = \tau T_L^* + J_M \dot{\omega}_M = \tau T_L^* + J_M \frac{\dot{\omega}_L}{\tau} \tag{5}$$

where:

$$T_L^* = T_L + J_L \dot{\omega}_L \tag{6}$$

is the generalized resistant torque at the load shaft. In Eq. (6) all the terms related to the load are known.

The root mean square value of the torque T_M is computed from Eqs. (1) and (5):

$$T_{M,rms}^2 = \int_0^{t_a} \frac{1}{t_a}\left(\tau T_L^* + J_M \frac{\dot{\omega}_L}{\tau}\right)^2 dt$$
$$= \tau^2 T_{L,rms}^{*2} + J_M^2 \frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2J_M (T_L^* \dot{\omega}_L)_{mean} \tag{7}$$

Introducing Eqs. (7) in (2) and dividing by the motor momentum of inertia J_M:

$$\frac{T_{M,N}^2}{J_M} \geqslant \tau^2 \frac{T_{L,rms}^{*2}}{J_M} + J_M \frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2(T_L^* \dot{\omega}_L)_{mean} \tag{8}$$

Now two parameters can be defined: the motor's accelerating factor α and the load factor β:

$$\alpha = \frac{T_{M,N}^2}{J_M} \tag{9}$$

$$\beta = 2\left[\dot{\omega}_{L,rms} T_{L,rms}^* + (\dot{\omega}_L T_L^*)_{mean}\right] \tag{10}$$

The coefficient α does not depend on the machine task, it's easy to calculate from manufacturer catalogs and can be traced back to the quantities used in [7–9,15].

On the contrary, the coefficient β depends only on the working conditions because it represents the power rate required by the system. The measurement unit of both factors is (W/s). Substituting α and β in Eq. (8):

$$\alpha \geqslant \beta + \left[T_{L,rms}^*\left(\frac{\tau}{\sqrt{J_M}}\right) - \dot{\omega}_{L,rms}\left(\frac{\sqrt{J_M}}{\tau}\right)\right]^2 \tag{11}$$

Since the term in brackets is always positive, or null, the load factor β represents the minimum value of the right hand side of Eq. (11).

2.3. Range of suitable transmission ratio

Solving the biquadratic inequality (11), for each motor, there is a range of acceptable gear ratios:

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T_{L,rms}^*}\left[\sqrt{\alpha - \beta + 4\dot{\omega}_{L,rms} T_{L,rms}^*} \pm \sqrt{\alpha - \beta}\right] \tag{12}$$

With:

$$\tau_{min} \leqslant \tau \leqslant \tau_{max} \tag{13}$$

the condition expressed in Eq. (2) is satisfied. The range width $\Delta\tau$ is a function of the difference between the two factors α and β:

$$\Delta\tau = \tau_{max} - \tau_{min} = \frac{\sqrt{J_M}}{T_{L,rms}^*}\sqrt{\alpha - \beta} \tag{14}$$