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Iterative solution to twin image problem in in-line digital holography

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ABSTRACT

An iterative approach based on phase retrieval and combination of constraints is proposed to reconstruct object wavefront free from twin image disturbances. Comparing with other iterative methods, the proposed method has a better elimination effect and faster rate of convergence. Both simulation and experimental results are presented to validate the feasibility of the proposed approach.

1. Introduction

In-line holography, also named "Gabor holography" after its inventor [1], is a well-established lensless imaging technique. The hologram is formed by the interference between the non-diffracted light as the role of reference beam and the diffracted components generated by the sample. Owing to the simplicity of its recording set-up, in-line holography has been widely used in numerous applications [2–6], especially suitable for short wavelengths [7,8]. While in the reconstruction process, the superposition of conjugate images and dc-term cannot be spatially separated as off-axis holography [9,10]. The unwanted components manifest as interference pattern, which blurs the details and degrades the contrast of reconstruction image.

In the past decades, various strategies have been proposed to eliminate the twin image in in-line holography using multiple holograms, i.e., phase shifting [11,12], multiple wavelengths [13,14], different recording planes [15,16], digital decoding [17,18] and subtraction [19–21]. However, these methods have different drawbacks, i.e., precise instruments [11–14], singularity problem [17], sensitive to the beam fluctuation and uneven of recording media [18], only restore real part of the complex object [19], weak object beam approximation [20], unable for investigation of spontaneous phenomenon [11–15] or limitation on the object size [21]. Linear filtering can be applied for single hologram, but this approach was only applicable to real objects [22]. Phase retrieval provides another solution for twin image elimination using single hologram [23–26]. By iteratively applying certain constraints, i.e., finite transmission or finite support, on an estimate of object in the real and the reciprocal domain, real image is well separated from the unwanted conjugate image. Liu performed the first investigation of using phase retrieval [24]. However, this algorithm is limited to purely absorbing objects and cannot recover phase shifts caused by transparent objects. Koren's approach utilizing iterations between in-focus and out-focus image domain is applicable for small objects with strong transmittance contrast [25]. Latychevskaia et al. used finite transmission constraint in their algorithm, which works well for the sample with a simple structure and weak phase variations [26].

In this paper, we propose a twin image elimination method based on phase retrieval using a single in-line hologram. A loose support is created from the conventional reconstruction by the Sobel edge detection [27]. It is adopted as one of the constraints at the object plane. For samples whose phase shifting are limited comparing with corresponding illuminating wavelength or recorded in short wavelength range, i.e., hard or soft X-ray [7,8,28], the positivity can be used as another constraint at the object plane. The magnitude of the normalized hologram is used as the constraint at the recording plane. By conducting propagation back and forth between the two planes, the twin image fades away and the reconstructed distribution reaches its true value. In Sections 2 and 3, we describe the formation of in-line hologram and give the formulas of the proposed approach. We present the simulation comparison results in Section 4 and experimental verification in Section 5. Finally, Section 6 offers concluding remarks.

2. In-line hologram formation

To fix the notation and vocabulary, the formation of in-line hologram is briefly reviewed. A finite-spatial-extent plane wave

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$A\exp\left(\frac{j2\pi z}{\lambda}\right)$ with amplitude A and wavelength λ propagates from object plane ($z=0$) towards the recording plane ($z=d$) where the illuminating intensity is $|A\exp\left(\frac{j2\pi d}{\lambda}\right)|^2=|A|^2=B$, thus provides a coherent background B. If an object is placed into the beam (at position $z=0$), the wavefront at the object plane can be represented as $U_r(\xi,\eta)$. The Huygens–Fresnel principle is applied to determine the scalar field distribution $U_h(x,y)$ at the recording plane (at position $z=d$). The field is given by the Fresnel diffraction formula [29]

$$U_h(x,y)=\frac{A}{j\lambda d}\exp(jkd)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U_r(\xi,\eta)\times\exp\left\{jk[(x-\xi)^2+(y-\eta)^2]/2d\right\}d\xi d\eta. \quad (1)$$

where $k=2\pi/\lambda$, (ξ,η) and (x,y) are the coordinates at the object plane and recording plane, respectively. The propagation between the two planes is simplified as:

$$U_h(x,y)=A\exp(jkd)F_d^+[U_r(\xi,\eta)],$$
$$U_r(\xi,\eta)=A^{-1}\exp(-jkd)F_d^-[U_h(x,y)], \quad (2)$$

where the operator $F_d^+[]$ and $F_d^-[]$ denote convolution and deconvolution with the propagation function, d is the propagation distance, and $+$ or $-$ refers to forward or backward propagation, respectively, along the Z axis.

The hologram can be represented as:

$$H(x,y)=U_h(x,y)U_h^*(x,y) = |A|^2|[U_r(x,y)]\times h_d(x,y)|^2. \quad (3)$$

By dividing the hologram image over the background image $H(x,y)/B(x,y)$, the normalized hologram is independent of $|A|^2$, thus the influence of light intensity, camera sensitivity, image intensity scale is minimized [30].

3. The algorithm to eliminate twin image

The aim of the proposed algorithm is to reconstruct $U_r(\xi,\eta)$ in the object plane. The direct reconstruction from the normalized hologram is used as the estimated initial guess to start the iteration. The propagation back and forth between the object and hologram domains is via convolution operation [29]. For object whose phase shift is less than a quarter of the illumination wavelength or illuminated by short wavelength, the constraint on the object domain is nonnegative for both real and imaginary part inside the object support, and the square root of normalized hologram is the magnitude constraint in the recording domain. The flowchart of the iterative procedure is shown in Fig. 1. It consists of following steps:

(a) Using the direct reconstruction $\hat{U}_r^1(\xi,\eta)$ from the normalized hologram as the initial guess.
(b) The boundary of a loose support is obtained with the combination of object edge detection and shape estimation, the former is obtained from calculation of intensity slopes in the direct reconstruction using a Sobel edge detector [27]:

$$\partial S(\xi,\eta)=\alpha,\ \alpha\in\left\{|grad|\hat{U}_r^1(\xi,\eta)|^2|\right\}, \quad (4)$$

where $S(\xi,\eta)$ denotes the loose support region, $\partial S(\xi,\eta)$ denotes the corresponding boundary. In the proposed method, the profile of the support is based on the edge detection of conventional reconstruction image. The shape estimation is based on the pre-knowledge of the object, i.e., inside or outside the calculated support.
(c) According to the definition of normalized hologram, the transmission function outside the object support S is equal to 1,

$$G[\hat{U}_r^n(\xi,\eta)]=\begin{cases}\hat{U}_r^n(\xi,\eta) & \text{if } (\xi,\eta)\in S\\ 1 & \text{otherwise.}\end{cases} \quad (5)$$

(d) The real and imaginary part inside the support region S should be nonnegative, the negative pixels are due to the interference between twin image and reference wave. However, directly setting these pixels to zero is too strong to reach global minimum and possibly bring stagnation in the iterations, especially in the presence of noise. Therefore, instead of applying a hard projection operation inside the support S, a relaxation parameter β [23] is introduced

$$\hat{U}_r^{n'}(\xi,\eta)=C[\hat{U}_r^n(\xi,\eta)]\ (\xi,\eta)\in S \quad (6)$$

where n denotes the number of iteration, the operator C is defined as

$$C[\hat{U}_r^n(\xi,\eta)]=\begin{cases}\hat{U}_r^n(\xi,\eta)-\beta\times\hat{U}_r^n(\xi,\eta) & \text{if } n=1\\ \hat{U}_r^{n-1}(\xi,\eta)-\beta\times\hat{U}_r^n(\xi,\eta) & \text{if } n\geq 2.\end{cases} \quad (7)$$

The relaxation parameter β is real, and ranges in $0<\beta<1$. If a value of $\hat{U}_r^n(\xi,\eta)$ inside support region remains negative due to the interference between the twin image and the reference wave, the corresponding point of $\hat{U}_r^{n'}(\xi,\eta)$ grow larger during the iterations until all points inside S go nonnegative [23].
If the object is complex valued, this constraint needs to be applied to real and imaginary part, respectively. If the object is real valued, this constraint is only applied to the real part.
(e) $\hat{U}_r^{n'}(\xi,\eta)$ propagates forward to form a new complex field $\hat{U}_h^n(x,y)$ at the recording plane. The phase obtained from propagation is extracted and combined with the square root of the normalized hologram. The updated complex amplitude at the recording plane is

$$\hat{U}_h^{n+1}(x,y)=T[\hat{U}_h^n(x,y)], \quad (8)$$

where the operator T is defined as

$$T\left[\hat{U}_h^n(x,y)\right]=\sqrt{\frac{H(x,y)}{B(x,y)}}\frac{\hat{U}_h^n(x,y)}{|\hat{U}_h^n(x,y)|}. \quad (9)$$

The transmission function $\hat{U}_r^{n+1}(\xi,\eta)$ is updated at the object plane using Eq. 2. The procedure is repeated from (c) to (e) to form a loop of a concatenation of operators,

$$\hat{U}_r^{n+1}(\xi,\eta)=CGF_d^-TF_d^+[\hat{U}_r^n(\xi,\eta)],$$
$$\hat{U}_h^{n+1}(x,y)=TF_d^+CGF_d^-[\hat{U}_h^n(x,y)]. \quad (10)$$

4. Results of the numerical simulation

To show the feasibility of the proposed algorithm, we conducted a simulation using a synthetic 2D complex object in Fig. 2(a). The following parameters were used in the process: $\lambda=532$ nm, $d=30$ mm, pixel pitch was 20 μm, the object size was 128×128 pixels, the size of illuminating planar wave and CCD were 512×512 pixels. β was equal to 0.9 in Eq. 9. The hologram was simulated using the propagation integral and displayed in Fig. 2(b). The profile of the support is a 128×128 pixels square [see Fig. 2(c)].calculated from the initially reconstructed distribution [see in Fig. 2(d)] by edge detection [27]. Comparing with the original distributions [see Fig. 2(a)], it is clear that the object