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Generalized phase-shifting interferometry by parameter estimation with the least squares method

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A simple non-iterative algorithm for generalized phase-shifting interferometry is proposed. This algorithm recovers the wrapped phase from two or more interferograms with unknown phase steps between 0 and π rad. The proposal is based on the least squares method to calculate four parameters: background and modulation light, phase steps and wrapped phase distribution. This algorithm, by a new interferogram normalization procedure, can handle interferograms with variable spatiotemporal visibility overcoming the restriction and drawbacks from usual variable spatial visibility approaches. The algorithm works very well for processing interferograms which include many fringes. This behaviour will be explicated and discussed. The effectiveness and robustness of this algorithm are supported by numerical simulation and by the evaluation of experimental interferograms. The phase-shift estimation quality is verified by two different techniques. By the properties of this algorithm, such as the low computing time and free of user intervention, we believe it could be used in automatic real-time applications.

1. Introduction

Optical interferometry is one of the most powerful tools used in optical testing [1]. In general, by using this tools we obtain fringe-patterns (interferograms) that have a phase distribution which contains the desired information such as the wavefront deformations. There are several methods for extracting the phase distribution from interferograms which can be divided into two broad categories: spatial methods and temporal (phase-shifting interferometry (PSI)) methods [2,3].

Spatial methods require that interferograms correspond to wavefronts with a carrier, i.e. tilted phase distribution such that the wavefront is monotonically increasing [4]. Such interferograms are characterized by having only open fringes. Then interferograms with closed fringes are ruled out of conventional spatial methods. In contrast, PSI methods work with any type of interferograms including open and closed fringes in any combination. In this case, a set of interferograms is required where the recorded phase between two interferograms change only by a constant phase step.

The standard PSI requires that all phase steps are equal [5–7]. However, this requirement is often difficult to fulfil due to many practical issues with miscalibration, non-linear response of piezo-electric device, and another perturbations [8]. To eliminate this inconvenience, the standard PSI has been extended by alternative approaches such as generalized data reduction and least squares algorithm [9,10]. However, in these cases the phase-shift must still be known *a priori*, although they do not require any special value. To overcome these drawbacks, the so called Generalized PSI (GPSI) in which the phase-shift is a to be determined unknown was proposed.

Perhaps, Lai and Yatagai [11] were the first to suggest a spatial method for phase-shift determination by using an additional optical setup to generate auxiliary Fizeau fringes. The additional optical setup was eliminated by including in the phase-shifted interferograms a carrier [12] or multiple carriers [13]. The carrier requirement has been relieved using different methods as the ellipse fitting method [14] from two or more interferograms with uniform visibility. With at least three interferograms, less severe restriction to the visibility can be reached by alternative approaches such as the two-dimensional Fourier–Hilbert demodulation [2], the direct global search stochastic algorithm [15,3], the cross-power spectrum and cross-bispectrum [16,17], and the hybrid formed by both principal component analysis and least squares method [18].

When at least five interferograms are available, the Lissajou ellipse fitting [19], or with at least 15, the max–min algorithm and the phase difference histogram [20–22] are useful. If as few as

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three or less interferograms are available but the computing time is not of concern, the iterative algorithms [23–25], the Fourier transform method [26], the windowed Fourier transform (WFT) [27], and the hybrid formed with the iterative and WFT approaches [28] are appropriate. Regardless of the computational cost, the WFT has found other applications such as the phase-shifter calibration [29], and the phase extraction from either phase-shifted interferograms, a single-carrier interferogram, or even a single closed-fringe pattern [30,31]. Also, for applications where the statistical nature of diffraction field is adequate, this can be exploited to phase-retrieval [32–34]; in this situation only two or more interferograms are required.

In this paper we present a simple faster non-iterative algorithm for GPSI that uses the least squares method to estimate four parameters: the background light, the modulation light, the phase steps, and the wrapped phase distribution. The former two parameters are necessary in order to normalize the interferograms. The interferogram normalization allows that the proposed algorithm can handle interferograms with visibility variable both spatially and temporally. This algorithm is totally automatic because it does not require user intervention, robust because it does not need noise filters, inexpensive computationally due to both the fact that it can work with only two or more interferograms and is non-iterative, and can extract any unknown phase step greater than zero and less or equal than π rad. These properties allow that the algorithm that we propose could be used in automatic real-time applications. Moreover, this algorithm has no restriction over the illumination profile because it can be represented by a high-order polynomial or by any appropriate surface as splines. In particular, in this work a second-degree polynomial (parabolic profile or second-order approximation of Gaussian profile) is used with the aim of making the algorithm's description simple and intuitive. The background and modulation light estimation is better when the interferograms have many fringes. This will be explicated and discussed later.

We will explain the algorithm's principles and then show its verification by computer simulation. We also carry out an optical experiment to test the suggested algorithm by processing real interferograms. The phase-shift estimation quality of the proposed algorithm is verified with the well-known ellipse fitting [14] and Fourier transform [26] methods.

2. Description of the algorithm

The algorithm that we are proposing has two parts. In the first one we normalize the interferograms by a new method where the background and modulation light parameters are estimated using the least squares method. In the second one, with the normalized interferograms, we calculate the phase steps to obtain the phase-shift and finally recovering the wrapped phase distribution.

2.1. Interferogram normalization

The interferogram normalization is the background and modulation light suppression. For this purpose, frequency analysis techniques have been successfully applied [35–39]; however, the efficiency is sensitive to filter design and can be computationally expensive. In this work we introduce a new interferogram normalization process based on polynomial surfaces fitting. This method exploits the information over the illumination profile, obtaining a faster and robust algorithm.

The normalization process implemented in this algorithm is based on two assumptions: first, the interferograms have a background illumination approximated to a parabolic profile, and second, the interferograms are formed by many fringes (open and closed fringes in any combination). The first assumption is a consequence of representing the general Gaussian illumination by using just a second-degree polynomial (a second order approximation). This is not a restriction because we can use a higher degree polynomial or another appropriate analytic bidimensional surface; however, here we are considering a second-degree polynomial in order to make a simple and intuitive description of the normalization procedure. The second assumption is needed so the cosine terms $b(p;k)\cos[\phi(p)+\delta_k]$ and $b^2(p;k)\cos[2\phi(p)+2\delta_k]/2$ from Eqs. (1) and (6), respectively, could be signals with zero mean; thus, an approximate solution for $a(p;k)$ and $b(p;k)$ can be estimated by a polynomial fitting procedure.

We consider a set of $K \geq 2$ interferograms. The k-th interferogram, with $k=0,1,\ldots,K-1$, can be defined as

$$I_k(p) = a(p;k) + b(p;k)\cos[\phi(p)+\delta_k], \tag{1}$$

where $p=(x,y)$ is a position vector with coordinates x and y in a rectangular domain given by $[x_1,x_M]\times[y_1,y_N]$ and $M\times N$ is the common size of all interferograms. The parameters a and b, functions of p and perhaps of k, defined as

$$a(p;k) = I_o(p;k) + I_r(p;k),$$
$$b(p;k) = 2[I_o(p)I_r(p)]^{1/2}$$

represents the background and modulation light, respectively. $I_o(p;k)$ is the object irradiance and $I_r(p;k)$ is the reference irradiance, $\phi(p)$ is the phase distribution to be measured, and the phase-shift δ_k is defined as

$$\delta_k := \begin{cases} 0 & \text{for } k=0, \\ \displaystyle\sum_{\ell=1}^{k} \alpha_\ell & \text{for } k>0, \end{cases} \tag{2}$$

where α_ℓ is the phase step from interferogram $I_{\ell-1}(p)$ to $I_\ell(p)$ with $\ell = 1,2,\ldots,N-1$. These phase steps α_ℓ can be solved for the interval $[0,\pi]$ rad; however, we assume that $0<\alpha_\ell\leq\pi$ in order to avoid singularity in Eq. (16). From Eq. (1), we can see that the functions $a(p;k)$, $b(p;k)$, δ_k, and $\phi(p)$ are the parameters of our model. The main parameter is $\phi(p)$ and we will obtain it by estimation of the remaining parameters. From now on, in the cases where there is no risk of confusion, the dependences of variables p and k are not written down for brevity.

Using a laser light source, the irradiances I_o and I_r are modeled by Gaussian functions. By Taylor expansion, the Gaussian profile is representable by a high-degree polynomial with degree equal to the order of approximation desired or, alternatively, by any appropriate analytic bidimensional surface, e.g. splines. In this sense, the proposed algorithm has no restriction over the illumination profile if a polynomial is used to represent it. In order to make a simple and intuitive description of this algorithm, we consider the particular case when a second-degree polynomial (parabolic profile or second-order approximation of Gaussian profile) is sufficient, namely $I_o \approx \mathcal{I}_o$ and $I_r \approx \mathcal{I}_r$, where $\mathcal{I}_o$ and $\mathcal{I}_r$ are second-degree polynomials. Thereby, $a \approx \mathcal{I}_o + \mathcal{I}_r$ is approximated to second-degree polynomial that we can write in a convenient notation as [1]

$$a \approx \sum_{u=0}^{2}\sum_{v=0}^{u} C_{uv}x^v y^{u-v}, \tag{3}$$

where $C_{uv}\in\Re$ are coefficients. As $\mathcal{I}_o$ and $\mathcal{I}_r$ are second-degree polynomials then the product $2\mathcal{I}_o\mathcal{I}_r \approx b^2/2$ is a fourth-degree polynomial:

$$\frac{1}{2}b^2 \approx \sum_{u=0}^{4}\sum_{v=0}^{u} D_{uv}x^v y^{u-v}, \tag{4}$$

where $D_{uv}\in\Re$ are coefficients.