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Propagation properties of partially coherent four-petal Gaussian vortex beams in turbulent atmosphere

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ABSTRACT

The partially coherent four-petal Gaussian vortex beam is introduced and described by analytical expressions. The analytical propagation equation for partially coherent four-petal Gaussian vortex beam in turbulent atmosphere is derived by using the extended Huygens–Fresnel diffraction integral formula. The influences of refraction index structure, beam order n, topological charge M and the coherence length on the average intensity distributions of beam are investigated by numerical examples.

1. Introduction

In recent years, the studies of propagation properties for various electromagnetic beams through turbulent atmosphere have the potential application in free-space optical communications and remote sensing [1]. Then, the propagation and evolution properties for laser beams through turbulent atmosphere have been widely investigated based on the extended Huygens–Fresnel diffraction integral formula [2–19]. For example, the average intensity of coherent laser beams, such as cosh-Gaussian beam, Hermite-sinusoidal-Gaussian beam, Hermite-cosine-Gaussian beam, dark hollow beam, flat-topped beam, vortex beam, Bessel vortex beam; and partially coherent beam, such as, electromagnetic stochastic beam, partially coherent multiple Gaussian beams, partially coherent radially polarized beam, elliptical Gaussian-Schell beam, random beam, partially coherent flat-topped beam, twisted Gaussian Schell-model beam and partially coherent dark hollow beam, propagating in turbulent atmosphere have been widely investigated.

With the development of laser optics, various laser beams have been introduced and investigated. In recent years, four-petal Gaussian beam has been introduced and investigated by researchers [20]. Since then, propagation properties of four-petal Gaussian beam have been studied [21–24]. However, to our knowledge, there have no reports about partially coherent four-petal Gaussian vortex beam, and the propagation properties of partially coherent four-petal Gaussian vortex beam through turbulent atmosphere.

In this paper, we first introduce the partially coherent four-petal Gaussian vortex beam based on the theory of coherence, then we investigate the average intensity of partially coherent four-petal Gaussian vortex beams propagating through turbulent atmosphere.

2. Propagation theory

The electric field of a four-petal Gaussian vortex beam propagating toward the half free space $z \geq 0$, and the z-axis is set to be the propagation axis, then the beam can be written as:

$$E(x_0, y_0, 0) = \left(\frac{x_0 y_0}{w_0^2}\right)^{2n} \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right)\left(x_0 + i\,\mathrm{sgn}(M) y_0\right)^{|M|} \tag{1}$$

where n denotes the order of the four-petal Gaussian beam, M is the topological charge of the spiral phase plate, w_0 denotes the waist width of Gaussian beam.

Based on the theory of coherence, the coherent four-petal Gaussian vortex beam can be extended to partially coherent beam. The second-order correlation properties of the electromagnetic beam can be characterized by the cross-spectral density matrix [25]:

$$\overleftrightarrow{W}(\mathbf{r}_1, r_2, z) = \left\langle E(\mathbf{r}_1, z) E^*(\mathbf{r}_2, z)\right\rangle = \sqrt{I(x_1, y_1, z) I(x_2, y_2, z)}\, g(x_1 - x_2, y_1 - y_2) \tag{2}$$

with $g(x_1 - x_2, y_1 - y_2)$ is the spectral degree of coherence assumed

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to have the Gaussian profile, and

$$g(x_1 - x_2, y_1 - y_2) = \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\sigma^2}\right] \tag{3}$$

where σ is the transversal coherence length of partially coherent four-petal Gaussian vortex beam.

Substituting Eq. (1) into Eq. (2), the element of cross-spectral density matrix for partially coherent four-petal Gaussian vortex beam can be written as

$$W_0(\mathbf{r}_{10}, \mathbf{r}_{20}, 0) = \left(\frac{x_{10}y_{10}}{w_0^2}\right)^{2n}\left(\frac{x_{20}y_{20}}{w_0^2}\right)^{2n}\exp\left(-\frac{r_{10}^2 + r_{20}^2}{w_0^2}\right)(x_{10} + i\,\mathrm{sgn}(M)y_{10})^{|M|} \times (x_{20} - i\,\mathrm{sgn}(M)y_{20})^{|M|}\exp\left[-\frac{(x_{10} - x_{20})^2}{\sigma^2} - \frac{(y_{10} - y_{20})^2}{\sigma^2}\right] \tag{4}$$

where $\mathbf{r}_{10} = (x_{10}, y_{10})$ and $\mathbf{r}_{20} = (x_{20}, y_{20})$ are the position vectors at the source plane $z=0$.

Within the framework of paraxial approximation, the average intensity of laser beams propagating through the turbulent atmosphere can be expressed as follows [1–5]:

$$\langle I(x, y, L)\rangle = \frac{k^2}{4\pi^2 z^2}\iiint\int_{-\infty}^{+\infty} W_0(\mathbf{r}_{10}, \mathbf{r}_{20}, 0) \times \exp\left[-\frac{ik}{2z}(r-\mathbf{r}_{10})^2 + \frac{ik}{2z}(r-\mathbf{r}_{20})^2\right] \times \left\langle \exp\left[\psi(x_{10}, y_{10}, x, y) + \psi^*(x_{20}, y_{20}, x, y)\right]\right\rangle dx_{10}dy_{10}dx_{20}dy_{20} \tag{5}$$

where $k = 2\pi/\lambda$ is the wave number; $\psi(x_0, y_0, x, y)$ is the solution to the Rytov method that represents the random part of the complex phase; the asterisk denotes the complex conjugation; $\mathbf{r} = (x, y)$ and $\mathbf{r}_0 = (x_0, y_0)$ are the position vectors at the output plane z and the input plane $z=0$, respectively. The ensemble average in Eq. (5) can be expressed as [5]:

$$\left\langle \exp\left[\psi(x_{10}, y_{10}, x, y) + \psi^*(x_{20}, y_{20}, x, y)\right]\right\rangle = \exp\left[-\frac{(x_{10} - x_{20})^2 + (y_{10} - y_{20})^2}{\rho_0^2}\right] \tag{6}$$

where ρ_0 is the spherical-wave lateral coherence radius due to the turbulence and

$$\rho_0 = \left(0.545C_n^2 k^2 z\right)^{-3/5} \tag{7}$$

with C_n^2 is the constant of refraction index structure and which describes the turbulence level.

By substituting Eq. (4) into Eq. (5), and recalling the following expansions [26]:

$$(x_0 + iy_0)^M = \sum_{l=0}^{M}\frac{M!i^l}{l!(m-l)!}x_0^{M-l}y_0^l \tag{8}$$

$$H_n(l) = \sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k n!}{k!(n-2k)!}(2l)^{n-2k} \tag{9}$$

$$\int_{-\infty}^{+\infty} x^n\exp\left(-px^2 + 2qx\right)dx = n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \tag{10}$$

After some tedious calculation we can obtain

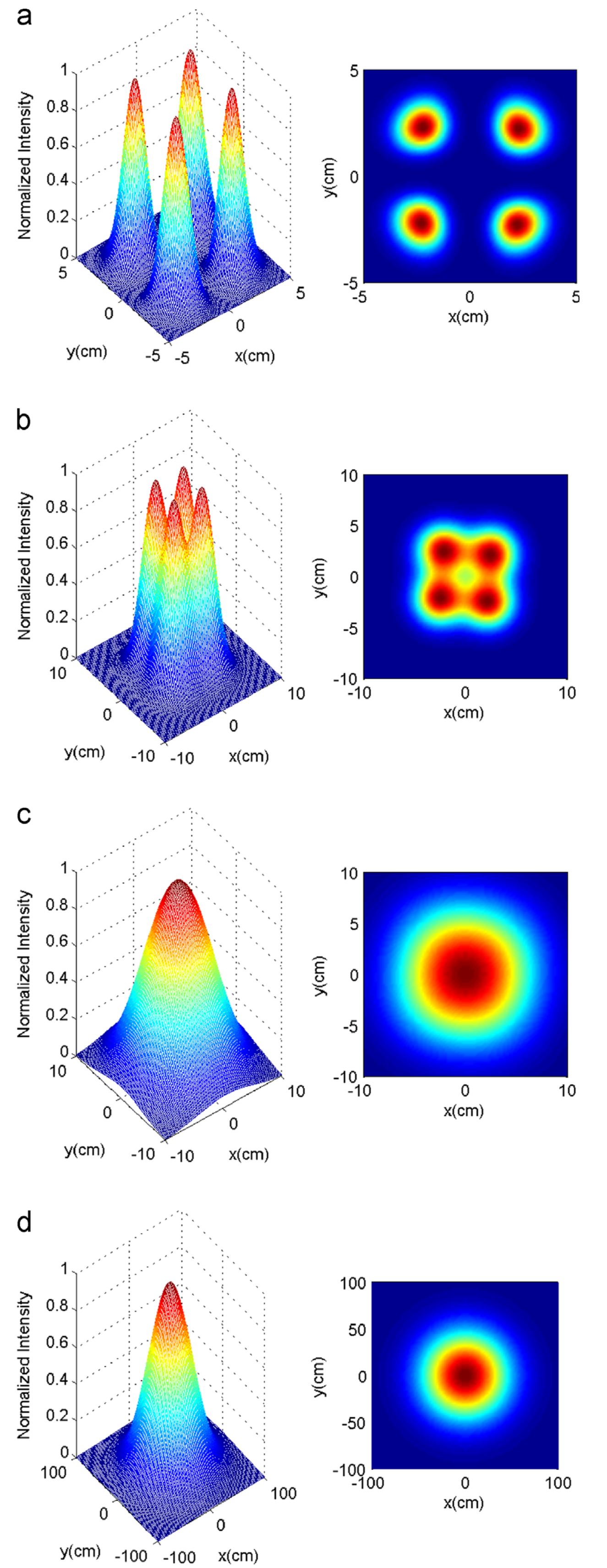

Fig. 1. The normalized average intensity of partially coherent four-petal Gaussian vortex beam propagating through turbulent atmosphere with $C_n^2 = 10^{-13}m^{-2/3}$, $n = 1$, $M=1$ and $\sigma = 10$ mm. (a) $z = 100$ m, (b) $z = 500$ m, (c) $z = 1000$ m, (d) $z = 5000$ m.