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An energy-based electroelastic beam model for MEMS applications

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Abstract

Competent modeling of an electroelastic beam configuration is of major importance for MEMS. MEMS motors, based on such a deformable capacitor, are driven by one or more voltage sources. A coherent model of limited complexity and expressed in design parameters is derived for an inchworm actuator. The two physical regimes; the stick and non-stick, are matched, such that a smooth, single energy profile arises. The pull-in voltage, the hysteresis loop, jump-back voltage, the step size, and the dissipated energies are determined as function of these design parameters. The results are compared to measurements with the μWalker.

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1. Introduction

A well-known effect when modeling and designing micro electromechanical systems (MEMS) is that the electrostatic forces scale favorably with decreasing size [1]. At the micrometer scale, an applied voltage in a range up to 100 V yields electrostatical forces comparable to the mechanical stiffness of the micro structure. This feature lies at the heart of an effective actuator, in our case an elastic beam with capacitance. The working principle of the elastic beam relies on a balance between electrostatic and mechanical forces. Step motors based on this principle, like the μWalker [2], are being developed for mass storage systems such as the μSPAM [3] and the Millipede [4]. As a result of electrostatic forces from an applied voltage difference, very fine and powerful steps are obtained. For the μWalker, the step size is a few tens of nanometers in size and by applying a high frequency of the walking cycle, velocities up to several mm/s are achievable with an accuracy in the order of 0.5 nm per step. Fig. 1 shows one realization of this micro device.

The characteristics of the beam greatly influence the properties of this device. For every applied voltage a static deformation exists. However, the corresponding dynamics is singular as above a critical pull-in voltage, the beam bending forces cannot compensate for the electrostatic forces and the beam will hit the underlying surface and stick there. Once the beam touches the ground plate, an area of contact is the result of a second and final balance between electrostatic and mechanical forces. A number of elaborate and detailed numerical investigations have been dedicated to this and similar systems. The cusp singularity [5] further complicates the analysis. We are not interested in the precise dynamics, and treat the system as singular: it jumps from one static state to another. A recent paper [6] does present a model for the pull-in voltage of a beam with fixed ends, whereas for the μWalker case one end moves freely. Furthermore, no stick region was taken into account in [6], which is essential to determine the step size, in the case of the μWalker. Especially the stick region is of major importance for hysteresis analysis and thus for calculating total energy losses and optimizing the design. A tractable, analytical model for both the electrostatic and elastic domains is derived. Various aspects of the beam motor are analyzed, such as hysteresis and design parameter dependencies. The beam inertia dynamics has been omitted

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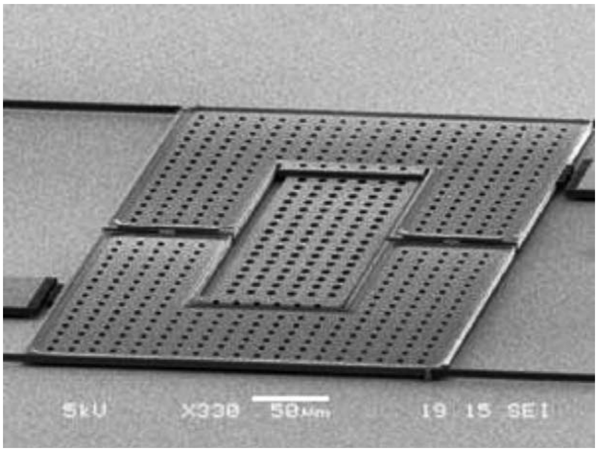

Fig. 1. One-dimensional version of the μWalker used for modeling and simulation (Courtesy of E. Sarajlic, TST Group, University of Twente).

here; mass effects and vibrations are not expected to have a major impact on the results. These effects can be coped with and implemented later on, in numerical studies of a refined model. Finally, the model is compared to measurements of the μWalker.

The working principle of the μWalker is based on the caterpillar's moving principle. Variation of the device output force is accomplished by changing the electrostatic forces applied. These forces are a result of voltage variations applied to the three distinct inputs, namely two for the supports and one for the beam. Let us assume that the device has to complete one step to the right. In the start position, only the right support is clamped by an applied voltage. The left support is free to move sideways. Next, the beam voltage is increased, such that the distance between the supports decreases by one step size. The left support is now clamped, while the right support is released shortly thereafter. The beam voltage is lowered and the beam relaxes. This completes one cycle of the μWalker. For more details, see Fig. 2.

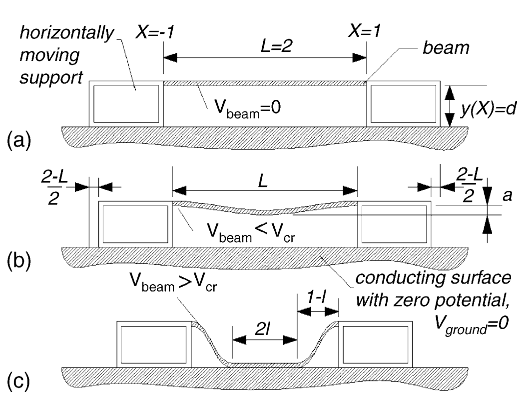

Fig. 2. The creeper beam is in either of the three modes: (a) at rest while no voltage applied ($V_{\mathrm{beam}} = 0$); (b) in free mode, due to an applied voltage below the critical voltage ($V_{\mathrm{beam}} < V_{\mathrm{cr}}$); (c) in stick mode due to a voltage V_{beam}, where $V_{\mathrm{beam}} > V_{\mathrm{cr}}$.

2. Elastic part

The system of a creeper beam has bending or elastic energy and electrostatic energy, which are treated in this and the next section, respectively. The beam profile is assumed to be a function of a single parameter for the beam deflection, a, where $a < d$ and d is the distance between the beam and the contact surface (see Fig. 2(a) and (b)). In a variational approach, where the minimum energy is determined for the beam profiles, the solution is an upper bound for the energy, and the typical pitfalls of nonlinear differential equations are avoided. The profiles are a function of a single parameter a, which is the distance from the rest position to the center of the beam, namely at $x = 0$:

$$y(x) = d - a(1 - x^2)^2, \tag{1}$$

where $y(x)$ is the vertical deflection, and $x \in [-1, 1]$ is the reference coordinate. Throughout the paper, the results are expressed in terms of the units of half of the length of the beam $L_0 = 2$, and energies are in units of energy per width of the beam w, since both the elastic and the electrostatic energy scale linearly with the width of the beam. In this way, the equations are easier to follow. Only at the end, the value of L_0 is set equal to the real beam length of the μWalker.

From numerical studies [7], the profile chosen has an appropriate shape, given the clamped boundary conditions. Even if small deviations of the beam shape occur, their influence on the energy is only of second order and negligible. The energy functional has an elastic and an electrostatic part which are determined below.

In simple beam models like the Euler–Bernoulli beam, only the vertical displacement y as a function of the horizontal rest, or reference coordinate x is determined. In our case, the horizontal displacement is also important for the walking motion. The length of the beam is taken to be fixed in the absence of stress through friction. Therefore, the horizontal position $X(x)$ as function of the reference coordinate x should satisfy:

$$1 = (\partial_x X(x))^2 + (\partial_x y(x))^2,$$

which is satisfied to second order in a by

$$\partial_x X(x) = 1 - \frac{1}{2}(\partial_x y(x))^2 = 1 - 8a^2(1 - x^2)^2 x^2,$$

where is assumed that $|1 - \partial_x X(x)| \ll |\partial_x y(x)|$, since $a \ll 1$. The horizontal distance L between the ends at $x = -1$ and $x = 1$ is therefore given by

$$L = \int_{-1}^{1} \partial_x X(x)\, \mathrm{d}x = 2 - \frac{128}{105} a^2. \tag{2}$$

The bending energy E_{b} of a fixed length beam is a function of the beam curvature k [8]: