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Constitutive relations for piezoelectric benders under various boundary conditions

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Abstract

In this article, the constitutive relations of three types of piezoelectric bender—the unimorph bender, bimorph bender and triple-layer bender—are derived based on beam theory under the quasi-static equilibrium condition. The relation coefficients are expressed in terms of the geometrical and material properties of the benders. Constitutive relations are derived for a range boundary conditions (fixed-free, fixed-roll and fixed-simply supported) under conditions allowing different lengths of the piezoelectric and elastic layers. The complicated constitutive relations can be easily formulated and verified by using the function for handling symbolic calculations. The derived constitutive relations may assist in the determination of the optimal values of geometrical parameters in the design of actuators and sensors for particular applications.

Keywords: Piezoelectric; Unimorph bender; Bimorph bender; Triple-layer bender; Quasi-static model

1. Introduction

The emergence of micro- and nano-machining in recent years has seen an upsurge in interest in piezoelectric transducers, which are essential components of micro-mechanical systems [1–3]. Piezoelectric transducers are devices that transform electrical energy into mechanical energy or vice versa [4]. For example, piezoelectric actuators can be used to force a micro-system to move to a desired position and piezoelectric sensors can be used to convert mechanical quantities such as acoustic pressure, force or position into electrical quantities such as voltage or charge. Piezoelectric benders can be used either as actuators that convert the longitudinal deformation induced by applying a voltage to a piezoelectric material into a bending displacement, or as sensors that convert a longitudinal deformation into a voltage. Piezoelectric benders can be classified as unimorph benders, bimorph benders, triple-layer benders and multilayer benders, depending on the number of layers and their arrangement. Unimorph benders consist of one piezoelectric layer and one nonpiezoelectric elastic layer. In these benders, a bending motion is produced by the different longitudinal deformation behavior of the piezoelectric and elastic layers. Bimorph benders are composed of two piezoelectric layers with opposite polarization directions such as a bimetal. Triple-layer benders, which were originally developed to enhance the mechanical reliability of piezoelectric benders, are made up of an elastic layer sandwiched between two piezoelectric layers.

In order to design piezoelectric benders suitable for a particular application, it is necessary to derive the piezoelectric constitutive equations for the system. These algebraic equations represent the correlations among the material properties, geometrical characteristics and physical parameters such as force, displacement, voltage and charge. However, derivation of the piezoelectric constitutive equations is a complex problem because piezoelectric materials have anisotropic and electromechanical coupling properties. Therefore, a simplified derivation of these equations is required for efficiently determining the optimal geometrical parameters of benders.

Numerous studies have examined the problem of formulating piezoelectric constitutive equations [5–17]; below we describe several representative examples. In the early 1990s, Smits et al. used a total internal energy method to derive unimorph and bimorph bender-constitutive equations with a 4×4 matrix form [5,6]. Subsequently, Cross

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Nomenclature

d_{ij}	piezoelectric constant
d_1	action point of force
d_2	action range of pressure
D_i	electric flux density
F_i	force
I_i	moment of inertia
$k^2 = d_{31}^2/\varepsilon_{33}^{\mathrm{T}} s_{\mathrm{p}}$	electromechanical coupling factor
L_i	length
M_i	moment
p	pressure
q	charge
$s_{11}^E = s_{\mathrm{p}} = 1/E_{\mathrm{p}}$	mechanical compliance of nonpiezoelectric material
$s_{\mathrm{m}} = 1/E_{\mathrm{m}}$	mechanical compliance of nonpiezoelectric material
t_i	thickness
V	voltage
w	width

Greek letters

α	angle
δ	displacement
$\varepsilon_{33}^{\mathrm{T}}$	dielectric constant
$\eta = s_{\mathrm{p}}/s_{\mathrm{m}} = E_{\mathrm{m}}/E_{\mathrm{p}}$	compliance (Young's modulus) ratio
$\lambda_1 = L_{\mathrm{p}}/L$	length ratio
$\lambda_2 = d_1/L$	forcing point with respect to length
$\lambda_3 = d_2/L$	pressing point with respect to length
ν	deflected volume
$\xi = t_{\mathrm{m}}/t_{\mathrm{p}}$	thickness ratio
$1/\rho_i$	radius of curvature

Subscripts

m	nonpiezoelectric (elastic) material
p	piezoelectric material

and coworkers derived the triple-layer bender constitutive equation using the method of Wang and Cross [7]. In other work, Devoe and Pisano derived a multimorph bender of cantilever type with one piezoelectric layer and several elastic layers, whose operating mechanism resembles that of a unimorph bender [8]. However, although, the equations derived in the above studies have been widely used in the design of micro-sensors and micro-actuators, they cannot be applied to benders with different layer lengths under boundary conditions other than those of cantilever type. Therefore, in the present study, we sought to derive more general constitutive relations involving fixed-free, fixed-roll and fixed-simply supported boundary conditions under the condition of nonidentical length of the piezoelectric and nonpiezoelectric layers. We adopted an approach using the superposition principle with geometrical compatibility constraints because the procedures for each formulation are considerably complicated and time-consuming. This approach makes it easier to formulate the complicated constitutive relations of piezoelectric bender. This approach also makes it possible to construct a computer program for obtaining closed forms of constitutive relations using the functions for symbolic calculations in the commercial package.

2. Derivation of the piezoelectric constitutive equation

In this section, the principles and assumptions used in deriving the piezoelectric constitutive relations are given. It is noted that the deformed state of benders by applying electric voltage can be obtained by applying on each layer of a bender the appropriate stretching (or compressing) forces and bending moments as illustrated in Fig. 1. In order to obtain the deformation by electric voltage application, the forces and bending torques should be appropriately determined such that the geometrical compatibility that strain in the interfaces between the layers is identical should be satisfied. The following assumptions are also made in order to simplify this approach:

(i) The stress distribution along the thickness direction is determined by superposing the stress due to bending on the uniformly-distributed stress due to one–three piezoelectric effects.
(ii) The bender acts as a pure Euler beam.
(iii) $\rho \gg t_{\mathrm{U}}, t_{\mathrm{L}}$, where ρ is the radius of curvature, and t_{U} and t_{L} are the thicknesses of the upper and lower layers.
(iv) There is no slip at the bonding surface.

In addition to the above assumptions, the desired relations among the curvature, moment and voltage can be developed by applying the following three principles.

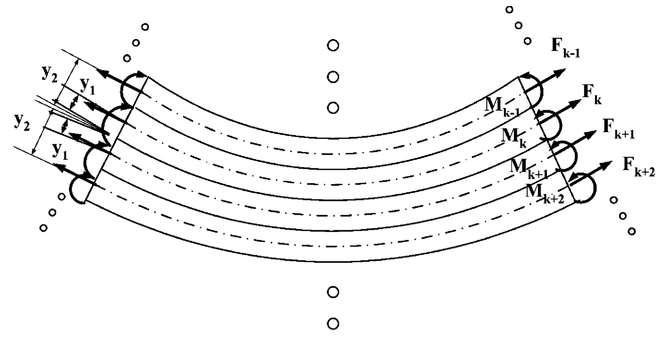

Fig. 1. Radius of curvature and external forces in the multilayer bender.