Available online at www.sciencedirect.com

SCIENCE DIRECT®

Solid-State Electronics 49 (2005) 923–927

SOLID-STATE ELECTRONICS

www.elsevier.com/locate/sse

Electron direct tunneling time in heterostructures with nanometer-thick trapezoidal barriers

Khairurrijal *, Fatimah A. Noor, Sukirno

Department of Physics, Institut Teknologi Bandung, Jalan Ganesa 10, Bandung 40132, Indonesia

Received 2 August 2004; received in revised form 23 February 2005; accepted 16 March 2005
Available online 22 April 2005

The review of this paper was arranged by Prof. S. Cristoloveanu

Abstract

An analytical expression of direct tunneling time of an electron through a nanometer-thick trapezoidal barrier has been derived. It is found that the direct tunneling time is independent of the thickness of the SiO_2 barrier for the SiO_2 layers thicker than 1 nm, which are used in advanced MOS (metal-oxide-semiconductor) devices. By applying a voltage to the barrier layer, the direct tunneling time becomes shorter than that obtained without the applied voltage. The nonparabolic energy-momentum dispersion of the barrier layer increases the direct tunneling time as compared to the parabolic one. However, the nonparabolic effect is negligible for high electron energy. It is also shown that the tunneling time obtained by the phase time is shorter than that calculated by the semiclassical approach for high electron energy. However, both of the calculated tunneling times have not been able to explain the reason why the calculated tunneling time are orders of magnitude shorter than the highest time resolution achievable in the silicon-based MOS devices.

Keywords: Direct tunneling time; Trapezoidal barrier; Wigner phase time; Nonparabolic dispersion

1. Introduction

Since introduced by Wigner [1] more than 40 years ago, the tunneling time of an electron through a barrier is still of interest in the study of quantum transport in heterostructures because the tunneling time is a key parameter for ultimate performance evaluations of resonant tunneling diodes, traveling-wave tunnel monolithic integrated circuits for picosecond applications, and infrared resonant tunneling lasers [2]. In recent years, the subject has received considerable attention in view of potential use of these structures in device fabrication such as a single barrier varactor diode for generating power up to submillimeter wave frequencies [3,4]. For a review of the literature on the tunneling time, see the recent article by Olkhovsky and Recami [5].

The Wigner phase time approach, which was proved to be the best model by Steinberg and Chiao [8], has been adopted by Lee [6] and Paranjape [7] in calculating electron direct tunneling time through a square potential barrier. However, they did not consider the effects of voltage applied to the barrier in which the square barrier becomes trapezoidal one.

In this paper, the electron direct tunneling time through a trapezoidal barrier is derived by employing the Wigner phase time. The results will provide us more precise information on the direct tunneling time and a basis of estimating heterostructure device operation frequencies.

* Corresponding author. Tel.: +62 22 2534161; fax: +62 22 2511848.
E-mail address: krijal@fi.itb.ac.id (Khairurrijal).

0038-1101/$ - see front matter
doi:10.1016/j.sse.2005.03.016

2. Theoretical model

The schematic energy diagram of a heterostructure with a voltage applied to the barrier is shown in Fig. 1. Here, the barrier width and height are L and Φ_0, respectively, V_b is the voltage applied to the barrier, and e is the electronic charge. We consider that the electron effective mass in region I is the same as that in region III (m_1) and in region II that is m_2.

We further consider the time-independent electron wave function in each region as [9]

$$\psi(z) = \begin{cases} A\exp(\mathrm{i}k_1 z) + B\exp(-\mathrm{i}k_1 z) & \text{for } z \leqslant 0, \\ C\exp\left(\int_0^z k_b(z')\,\mathrm{d}z'\right) & \\ \quad +D\exp\left(-\int_0^z k_b(z')\,\mathrm{d}z'\right) & \text{for } 0 < z < L, \\ G\exp(\mathrm{i}k_3 z) + H\exp(-\mathrm{i}k_3 z) & \text{for } z \geqslant L. \end{cases} \tag{1}$$

The incident wave $A\exp(\mathrm{i}k_1 z)$ has the energy

$$E = \frac{\hbar^2 k_1^2}{2m_1}, \tag{2}$$

where $\hbar$ is the reduced Planck constant and E is smaller than the barrier height Φ_0. The wave numbers $k_b(z)$ and k_3 are expressed as follows:

$$k_b(z) = \left\{\frac{2m_2(V(z) - E)}{\hbar^2}\right\}^{1/2}, \tag{3}$$

where $V(z) = \Phi_0 - eFz$ is the potential energy profile of the barrier, $F = V_b/L$ is the electric field in the barrier, and

$$k_3 = \left\{\frac{2m_1(E + eV_b)}{\hbar^2}\right\}^{1/2}. \tag{4}$$

With the boundary conditions at $z = 0$ and $z = L$, which are given by [10]

$$\psi(z = 0^-) = \psi(z = 0^+), \quad \frac{1}{m_1}\frac{\mathrm{d}\psi}{\mathrm{d}z}\bigg|_{z=0^-} = \frac{1}{m_2}\frac{\mathrm{d}\psi}{\mathrm{d}z}\bigg|_{z=0^+},$$
$$\psi(z = L^-) = \psi(z = L^+), \quad \frac{1}{m_2}\frac{\mathrm{d}\psi}{\mathrm{d}z}\bigg|_{z=L^-} = \frac{1}{m_1}\frac{\mathrm{d}\psi}{\mathrm{d}z}\bigg|_{z=L^+}, \tag{5}$$

it is easy to find the relation between the constants G and A in Eq. (1). The result can be written as

$$f \equiv \frac{G}{A} = \frac{2\frac{k_1 k_b^L k_b^L}{k_b^0}\left\{\left(k_3 + \frac{k_1 k_b^L}{k_b^0}\right)k_b^L\cosh u + \mathrm{i}\left(\frac{m_2 k_1 k_3}{m_1 k_b^0} - \frac{m_1 k_b^L}{m_2}\right)k_b^L\sinh u\right\}\exp(-\mathrm{i}k_3 L)}{\left\{\left(k_3 + \frac{k_1 k_b^L}{k_b^0}\right)k_b^L\cosh u\right\}^2 + \left\{\left(\frac{m_2 k_1 k_3}{m_1 k_b^0} - \frac{m_1 k_b^L}{m_2}\right)k_b^L\sinh u\right\}^2}, \tag{6}$$

where $u = \int_0^L k_b(z)\,\mathrm{d}z$. The magnitude of f is equal to

$$|f| = \frac{2\frac{k_1 k_b^L}{k_b^0}}{\sqrt{\left\{\left(k_3 + \frac{k_1 k_b^L}{k_b^0}\right)\cosh u\right\}^2 + \left\{\left(\frac{m_2 k_1 k_3}{m_1 k_b^0} - \frac{m_1 k_b^L}{m_2}\right)\sinh u\right\}^2}} \tag{7}$$

and the phase of f is given by

$$\phi = \tan^{-1}\left(\frac{\left(\frac{m_2 k_1 k_3}{m_1 k_b^0} - \frac{m_1 k_b^L}{m_2}\right)}{\left(k_3 + \frac{k_b^L k_1}{k_b^0}\right)}\tanh u\right) - k_3 L. \tag{8}$$

If the voltage applied to the barrier is zero ($F = 0$), then $k_3 = k_1$, $k_b^0 = k_b^L \equiv \gamma = (2m_2(\Phi_0 - E)/\hbar^2)^{1/2}$ and the expression of f is the same as that given by Lee [6]

$$f = \frac{2k_3\gamma\left\{2k_3\gamma\cosh\gamma L + \mathrm{i}\left(\frac{m_2}{m_1}k_3^2 - \frac{m_1}{m_2}\gamma^2\right)\sinh\gamma L\right\}\exp(-\mathrm{i}k_3 L)}{\{2k_3\gamma\cosh\gamma L\}^2 + \left\{\left(\frac{m_2}{m_1}k_3^2 - \frac{m_1}{m_2}\gamma^2\right)\sinh\gamma L\right\}^2}. \tag{9}$$

The direct tunneling time τ of an electron through the trapezoidal barrier is obtained by using the Wigner phase time approach [6].

$$\tau = \frac{m_1}{\hbar k_3}\left(\frac{\partial\phi}{\partial k_3} + L\right). \tag{10}$$

Substituting Eq. (8) into Eq. (10), we get

$$\begin{aligned}\tau = \frac{m_1}{\hbar k_3}&\frac{1}{\left\{\left(k_b^0 k_3 + k_b^L k_1\right)\cosh u\right\}^2 + \left\{(m_2 k_1 k_3/m_1) - \left(m_1 k_b^0 k_b^L/m_2\right)\sinh u\right\}^2} \\ &\times\left\{\frac{\hbar^2\left(k_b^L - k_b^0\right)}{efm_2}k_3\left(k_b^0 k_3 + k_b^L k_1\right)\left[\left(\frac{m_2}{m_1}\right)^2 k_1 k_3 - k_b^0 k_b^L\right] + \left[\frac{m_2}{m_1}\left(k_b^L k_1^2 + \frac{k_b^0 k_3^3}{k_1}\right) + \frac{\left(k_b^L\right)^2 k_1 k_3}{k_b^0} + \frac{\left(k_b^0\right)^2 k_3^2}{k_b^L}\right.\right. \\ &\left.\left. + \left(\frac{m_2}{m_1}\right)^2\left(\frac{k_1 k_3^3}{k_b^0} + \frac{k_1^2 k_3^2}{k_b^L}\right) + \frac{m_1}{m_2}\left(\left(k_b^0\right)^2 k_b^L + \frac{k_b^0\left(k_b^L\right)^2 k_3}{k_1}\right)\right]\frac{\sinh 2u}{2}\right\}.\end{aligned} \tag{11}$$