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On the C_0-semigroup generation and exponential stability resulting from a shear force feedback on a rotating beam

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Abstract

In this paper, we show that a linear unbounded operator associated with an Euler–Bernoulli beam equation under shear boundary feedback generates a C_0-semigroup in the underlying state Hilbert space. This provides an answer to a long time unsolved problem due to the lack of dissipativity for the operator. The main steps are a careful estimation of the Green's function and the verification of the Riesz basis property for the generalized eigenfunctions. As a consequence, we show that this semigroup is differentiable and exponentially stable, which is in sharp contrast to the properties possessed by most feedback controlled beams based on a passive design principle.

Keywords: Riesz basis; C_0-semigroup; Differentiable semigroup; Stability

1. Introduction

Let A be a densely defined closed linear operator in a Hilbert space H. The Hille–Yosida Theorem says that A generates a C_0-semigroup on H if and only if the following condition is satisfied:

$$\|R^n(\lambda, A)\| \leqslant \frac{M}{(\lambda-\omega)^n} \quad \forall \lambda > \omega \text{ and all integers } n \geqslant 1 \tag{1.1}$$

for some real numbers $M \geqslant 1$ and ω. This condition plays a significant role in the theoretical study of the C_0-semigroups because it is both a necessary and sufficient condition. As for applications, the Hille–Yosida Theorem

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is usually used in contraction semigroups or analytic semigroups because condition (1.1) can be replaced by one single requirement on the first power of the resolvent operator. When neither A is dissipative nor A generates an analytic semigroup, the theorem is usually hard to apply due to the difficulties in estimating all the nth powers of the resolvent operator. One of the remarkable results in overcoming this difficulty in recent years is to utilize the first power of the resolvent operator of A^*, the adjoint of A, and replaces condition (1.1) by [18]

$$\sup_{\sigma>\omega}(\sigma-\omega)\int_{-\infty}^{\infty}\|R(\sigma+\mathrm{i}\tau,\mathscr{A})x\|^2\,\mathrm{d}\tau<\infty\quad\forall x\in H;$$

$$\sup_{\sigma>\omega}(\sigma-\omega)\int_{-\infty}^{\infty}\|R(\sigma-\mathrm{i}\tau,\mathscr{A}^*)y\|^2\,\mathrm{d}\tau<\infty\quad\forall y\in H. \tag{1.2}$$

Unfortunately, even for condition (1.2), difficulties occur on some systems in estimating the integrals (1.2) in the infinite interval. All these difficulties are exactly why the issue of C_0-semigroup generation of the following problem, over these years, is still unsettled (see [14]).

In this article, we shall study an Euler–Bernoulli beam equation under a boundary shear feedback control:

$$\begin{aligned}
&y_{tt}(x,t)+y_{xxxx}(x,t)=0,\quad 0<x<1,\quad t>0,\\
&y(0,t)=y_x(0,t)=y_{xx}(1,t)=0,\\
&y_{xxx}(1,t)=u(t),\\
&y_{\text{out}}(t)=y_{xt}(1,t),\\
&u(t)=ky_{\text{out}}(t),
\end{aligned}\tag{1.3}$$

where $u(t)$ is the boundary control (input) imposed at the right end of the beam and $y_{\text{out}}(t)$ is the observation (output) measured at the same end. $k>0$ is the feedback gain constant that can be tuned in practice. This system describes the suppression of vibrations arose from the arm flexibility of a rotating Cartesian or SCARA robots with long arm in one direction. We refer readers to [14] for the detail physical background. Advantages of this shear feedback design have been explained numerically in [14]: the real parts of the eigenvalues of (1.3) would tend to $-\infty$ whereas the commonly used collocated feedback $y_{xxx}(1,t)=ky_t(1,t)$ would only tend to a vertical line parallel to the imaginary axis [10].

Another fact that may be interesting from the system control point of view is that the system (1.3) is not well-posed in the sense of Salamon–Weiss class [2]. This is because that the transfer function from $u(t)$ to $y_{\text{out}}(t)$ is given by

$$\hat{y}_{\text{out}}(s)=H(s)\hat{u}(s),\quad H(s)=-\mathrm{i}\frac{\sinh\tau\,\sin\tau}{1+\cosh\tau\,\cos\tau},\quad s=\mathrm{i}\tau^2, \tag{1.4}$$

where ˆ denotes the Laplace transform. Pick $\tau:=2n\pi+\pi/2$ for positive integer n. Then $|H(s)|=|\sinh\tau|\to\infty$ as $n\to\infty$. Hence the transfer function is unbounded in any right-half complex plane. That is, system (1.3) is not a Salamon–Weiss well-posed system.

Let the underlying energy Hilbert space for (1.1) be $\mathscr{H}=H_{\mathrm{E}}^2(0,1)\times L^2(0,1)$ with $H_{\mathrm{E}}^2(0,1)=\{f\in H^2(0,1)\mid f(0)=f'(0)=0\}$, and the inner product induced norm given by

$$\|(f,g)\|^2:=\int_0^1[|f''(x)|^2+|g(x)|^2]\,\mathrm{d}x\quad\forall(f,g)\in\mathscr{H}.$$