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Piecewise-affine state feedback for piecewise-affine slab systems using convex optimization

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Abstract

This paper shows that Lyapunov-based state feedback controller synthesis for piecewise-affine (PWA) slab systems can be cast as an optimization problem subject to a set of linear matrix inequalities (LMIs) analytically parameterized by a vector. Furthermore, it is shown that continuity of the control inputs at the switchings can be guaranteed by adding equality constraints to the problem without affecting its parameterization structure. Finally, it is shown that piecewise-affine state feedback controller synthesis for piecewise-affine slab systems to maximize the decay rate of a quadratic control Lyapunov function can be cast as a set of quasi-concave optimization problems analytically parameterized by a vector. Before casting the synthesis in the format presented in this paper, Lyapunov-based piecewise-affine state feedback controller synthesis could only be formulated as a bi-convex optimization program, which is very expensive to solve globally. Thus, the fundamental importance of the contributions of the paper relies on the fact that, for the first time, the piecewise-affine state feedback synthesis problem has been formulated as a convex problem with a parameterized set of LMIs that can be relaxed to a finite set of LMIs and solved efficiently to a point near the global optimum using available software. Furthermore, it is shown for the first time that, in some situations, the global can be exactly found by solving only one concave problem.

Keywords: Piecewise-affine systems; State feedback; Convex optimization; Ellipsoids; Lyapunov function

1. Introduction

Piecewise-affine systems are multi-model systems that offer a good modeling framework for complex dynamical systems involving nonlinear phenomena. In fact, many nonlinearities that appear frequently in engineering systems are either piecewise-affine (e.g., a saturated linear actuator characteristic) or can be approximated as piecewise-affine functions. Piecewise-affine systems are also a class of hybrid systems, i.e, systems with a continuous-time state

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and a discrete-event state. For piecewise-affine systems the discrete-event state is associated with discrete modes of operation. The continuous-time state is associated with the affine (linear with offset) dynamics valid within each discrete mode. Piecewise-affine systems pose challenging problems because of its switched structure. In fact, the analysis and control of even some simple piecewise-affine systems have been shown to be either an $\mathcal{NP}$ hard problem or undecidable [4].

State and output feedback control of continuous-time piecewise-affine systems have received increasing interest over the last years [10,13,15,21]. The interesting approach presented in [13,15] relies on computing upper and lower bounds to the optimal cost of the controller obtained as the solution to the Hamilton–Jacobi–Bellman equation. However, the continuous-time controller resulting from the approach in [15] is a patched LQR that cannot be guaranteed to avoid sliding modes at the switching and, therefore, is not provably stabilizing. Previous work of the authors has concentrated on Lyapunov-based controller synthesis methods for continuous-time piecewise-affine (PWA) systems [10,21]. In [21], Lyapunov-based controller synthesis was formulated as a bi-convex optimization problem. The bi-convexity structure arises because of the negativity constraint on the derivative of the piecewise-quadratic Lyapunov function over time. This constraint leads to a bilinear matrix inequality (BMI) [8]. Bi-convex optimization problems are non-convex, $\mathcal{NP}$ hard and, therefore, extremely expensive to solve globally from a computational point of view [8]. Based on this fact, Ref. [21] has adapted three alternative iterative algorithms for solving the non-convex problem to a suboptimal solution. Although the controller synthesis problem for piecewise-affine systems using piecewise-quadratic Lyapunov functions is non-convex, Hassibi and Boyd [10] have shown that for the particular case of piecewise-linear state feedback of slab piecewise-linear systems (without affine terms), globally quadratic stabilization could be cast as a convex optimization problem. Unfortunately, if affine terms are included in the controller, as stated in [10], "it does not seem that the condition for stabilizability can be cast as an LMI", which apparently destroys the convex structure of the problem, making it hard to solve globally. The current paper shows that piecewise-affine state feedback for piecewise-affine slab systems using a globally quadratic Lyapunov function can indeed be solved to a point near the global optimum in an efficient way by a set of LMIs. Building on the result of [10], this paper formulates piecewise-affine state feedback as an optimization problem involving a set of LMIs analytically parameterized by a vector. Three different algorithms will be suggested to solve relaxations of the optimization problem to a point near the global optimum. One is based on gridding of the domain of the parameterizing vector and yields solutions that approach the global optimum as the density of the grid is increased. The others are based on trace maximization to approximately solve an LMI subject to rank constraints [17], a problem that appears frequently in reduced order controller design. Although yielding solutions approaching the global optimum, the algorithm involving gridding increases the computational cost as the grid becomes denser and can be prohibitive for large systems. However, the gridding approach has already been used in other recent research on analysis [9], LPV control [23], gain-scheduling control [1,2] and some techniques already exist to alleviate the computational cost due to the gridding phase [3,22]. The algorithms for trace maximization are inspired by the work presented in [11,7,6]. One of these algorithms is iterative but typically involves only one or two iterations, thus typically being less computationally expensive than the gridding algorithm. The other proposed trace maximization algorithm is simply a concave program, which is therefore efficient from a computational point of view. It is also shown in the paper that constraints for continuity of the control inputs can be added to the PWA state feedback problem without affecting its parameterization structure. Finally, it is shown that piecewise-affine state feedback controller synthesis for piecewise-affine slab systems to maximize the decay rate of a globally quadratic control Lyapunov function can be cast as a set of quasi-concave optimization problems analytically parameterized by a vector. This problem can also be solved numerically using efficient algorithms.

In this paper, four controller synthesis problems are formulated, relaxed to a finite set of convex optimization problems and solved. The paper starts by presenting the assumptions that are common to all controller design problems, followed by the statements of the four problems. Section 4 formulates the controller synthesis problems as optimization programs. Section 5 presents several algorithms to solve the formulated problems. Finally, after two numerical examples, the paper finishes by presenting the conclusions