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A note on strict passivity

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Abstract

We show that there exists an explicit descriptor state space format which actually describes all strictly passive transfer functions. A key advantage of this explicitly strictly passive descriptor state space format resides in its relation with congruence projection-based reduced order modeling, where the resulting reduced order model is also cast in this same format. Another advantage of the format is that it allows for a simple construction of strictly passive random systems generators.
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1. Introduction

For time-invariant linear dynamical systems, strict passivity guarantees stability and the possibility of synthesis of the transfer function by means of a lossy physical network of resistors, capacitors, inductors and transformers [1]. It is well-known that strict passivity is equivalent with the strict positive reality of the system's transfer function [3]. Hence the strict passivity of a linear system can be checked by determining whether its transfer function is strictly positive real, and this in turn, by the well-known Kalman–Yakubovich–Popov positive-real lemma, implies testing the solvability of certain linear matrix inequalities (LMIs). It is known [3] that there are explicit solutions to LMI problems for only a few very special cases. However, they can be solved numerically by interior point methods.

In this paper we tackle the strictly positive real problem in another fashion. We show that there exists an explicit descriptor state space format involving positive definite matrices, which actually describes all strictly positive real transfer functions. One of the main advantages of this explicitly strictly passive descriptor state space format resides in its connection with congruence projection-based reduced order modeling, where the resulting reduced order model is cast directly in the same strictly passive state space format. Another advantage is that it allows for a simple construction of a strictly passive random systems generator.

2. Main results

In what follows X^{T} and X^{H}, respectively, denote the transpose and Hermitian transpose of a matrix X, and I_m denotes the identity matrix of dimension m. For two

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Hermitian matrices X and Y, the matrix inequalities $X > Y$ or $X \geqslant Y$ mean that $X - Y$ is, respectively, positive definite or positive semidefinite. For the real system with minimal realization

$$\dot{x} = Ax + Bu, \tag{1}$$

$$y = L^{\mathrm{T}}x + Du, \tag{2}$$

where $B \neq 0$ and $L \neq 0$ are $n \times p$ real matrices and A is a $n \times n$ real matrix, to be strictly passive (also called strictly positive real), it is required that the $p \times p$ transfer function

$$H(s) = L^{\mathrm{T}}(sI_n - A)^{-1}B + D \tag{3}$$

is analytic in the open right halfplane $\Re[s] > 0$, such that

$$H(i\omega) + H(i\omega)^{\mathrm{H}} \geqslant \varepsilon I_p \quad \forall \omega \in \mathbb{R} \tag{4}$$

for some $\varepsilon > 0$. This naturally implies that all the poles of $H(s)$ must be located in the open left halfplane $\Re[s] < 0$, or stated otherwise: A must be stable, i.e. $\Re[Sp(A)] < 0$.

Note that, from requirement (4), it is readily seen that adding a constant $p \times p$ matrix D_0 to a merely passive $H(s)$ results in a strictly passive transfer function $H(s) + D_0$ if and only if $D_0 + D_0^{\mathrm{T}} > 0$. Before proving our main result we need the following

Lemma. *Let*

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^{\mathrm{T}} & M_{22} \end{bmatrix} \tag{5}$$

be a $(n+p) \times (n+p)$ symmetric matrix partitioned in its $n \times n, n \times p, p \times n, p \times p$ blocks. Then $M > 0$ if and only if there exists a $n \times n$ nonsingular matrix Q and a $n \times p$ matrix W such that

$$M_{11} = QQ^{\mathrm{T}},$$

$$M_{12} = QW,$$

$$M_{22} > W^{\mathrm{T}}W. \tag{6}$$

Proof. Let Q and W satisfy (6). Then M can be written as

$$M = \begin{bmatrix} Q & 0 \\ W^{\mathrm{T}} & I_p \end{bmatrix} \begin{bmatrix} I_n & 0 \\ 0 & M_{22} - W^{\mathrm{T}}W \end{bmatrix} \begin{bmatrix} Q^{\mathrm{T}} & W \\ 0 & I_p \end{bmatrix}. \tag{7}$$

Since $M_{22} - W^{\mathrm{T}}W > 0$, the matrix M is a congruence of a positive definite matrix and hence itself positive definite.

Conversely, if $M > 0$ then $M_{11} > 0$ and hence has a Cholesky factorization $M_{11} = QQ^{\mathrm{T}}$. Now, with $W = Q^{-1}M_{12}$ it is evident that (7) is a block Cholesky factorization of M and hence $M_{22} - W^{\mathrm{T}}W > 0$ must hold. □

Theorem 1. *Let system* (1)–(2) *with transfer function*

$$H(s) = L^{\mathrm{T}}(sI_n - A)^{-1}B + D \tag{8}$$

be strictly passive (*and hence stable*). *Then there exists a $n \times n$ matrix $P = P^{\mathrm{T}} > 0$, a $n \times n$ matrix G such that $G + G^{\mathrm{T}} > 0$ and a $n \times p$ matrix R such that $H(s)$ can be written as*

$$H(s) = L^{\mathrm{T}}(sP + G)^{-1}R + \tfrac{1}{2}(L - R)^{\mathrm{T}}(G + G^{\mathrm{T}})^{-1} \times (L - R) + D_1, \quad D_1 + D_1^{\mathrm{T}} > 0. \tag{9}$$

Conversely, let $P = P^{\mathrm{T}} > 0$ and G such that $G + G^{\mathrm{T}} > 0$. Then the system with transfer function (9) *is strictly passive.*

Proof.

- *Direct part of the theorem*: It is known [3] that requirement (4) is satisfied if and only if there exists a $n \times n$ symmetric matrix $P = P^{\mathrm{T}} > 0$ satisfying the LMI

$$\begin{bmatrix} A^{\mathrm{T}}P + PA & PB - L \\ B^{\mathrm{T}}P - L^{\mathrm{T}} & -D - D^{\mathrm{T}} \end{bmatrix} < 0. \tag{10}$$

By the Lemma, this is equivalent with finding P, a $n \times n$ nonsingular matrix Q and a $n \times p$ matrix W such that

$$A^{\mathrm{T}}P + PA = -QQ^{\mathrm{T}} < 0, \tag{11}$$

$$PB - L = -QW, \tag{12}$$

$$D + D^{\mathrm{T}} > W^{\mathrm{T}}W \geqslant 0. \tag{13}$$

After eliminating Q and W we obtain the inequality

$$D + D^{\mathrm{T}} > -(L - PB)^{\mathrm{T}} \times (A^{\mathrm{T}}P + PA)^{-1}(L - PB). \tag{14}$$

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