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Composite nonlinear control with state and measurement feedback for general multivariable systems with input saturation

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Abstract

In this paper, we present a design procedure of composite nonlinear feedback control for general multivariable systems with actuator saturation. We consider both the state feedback case and the measurement feedback case without imposing any restrictive assumption on the given systems. The composite nonlinear feedback control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with faster rise time, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to reduce overshoot and undershoot caused by the linear part. As such, a highly desired tracking performance with faster settling time and smaller overshoot can be obtained. The result is illustrated by a numerical example, which shows that the proposed design method yields a very satisfactory performance.

Keywords: Nonlinear control; Actuator saturation; Tracking control

1. Introduction and problem formulation

Every physical system in our real life has nonlinearities and very little can be done to overcome them. Many practical systems are sufficiently nonlinear so that important features of their performance may be completely overlooked if they are analyzed and designed through linear techniques (see e.g., [8]). When the actuator is saturated, the performance of the control system designed will seriously deteriorate. As such, the topic of nonlinear control for saturated linear systems has attracted considerable attentions in the past (see e.g. [6,7,10,12,13,16] to name a few).

Inspired by a work of Lin et al. [9], which was introduced to improve the tracking performance under state feedback laws for a class of second-order SISO systems subject to actuator saturation, Chen et al. [3] have recently extended the so-called composite nonlinear feedback (CNF) control technique to general SISO systems with measurement feedback. The work

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of Chen et al. [3] has been successfully applied to design an HDD servo system. It has also been demonstrated in [3] that the CNF design is capable of beating the time-optimal control in asymptotically tracking situations. The extension of the result of [9] to MIMO systems under state feedback was reported in [15]. However, the extension was made under a pretty odd assumption, which will be discussed later.

In this paper, we present a design procedure of the CNF control for improving tracking performance of general multivariable systems with actuator saturation. Generally, the CNF control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with faster rise time, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to reduce overshoot and undershoot caused by the linear part. More specifically, we consider a multivariable linear system Σ with an amplitude-constrained actuator characterized by

$$\begin{aligned} \dot{x} &= Ax + B\,\mathrm{sat}(u), \quad x(0) = x_0, \\ y &= C_1 x, \\ h &= C_2 x + D_2\,\mathrm{sat}(u), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $h \in \mathbb{R}^\ell$ are, respectively, the state, control input, measurement output and controlled output of the given system Σ. A, B, C_1 and C_2 are appropriate dimensional constant matrices, and the saturation function is defined by

$$\mathrm{sat}(u) = \begin{pmatrix} \mathrm{sat}(u_1) \\ \mathrm{sat}(u_2) \\ \vdots \\ \mathrm{sat}(u_m) \end{pmatrix},$$
$$\mathrm{sat}(u_i) = \mathrm{sign}(u_i)\min(|u_i|,\ \bar{u}_i), \tag{2}$$

where $\bar{u}_i$ is the maximum amplitude of the ith control channel. The objective of this paper is to design an appropriate control law for (1) using the CNF approach such that the resulting controlled output will track some desired step references as fast and as smooth as possible. We will address the CNF control system design for the given system (1) for three different situations, namely, the state feedback case, the full order measurement feedback case and the reduced order measurement feedback case. For tracking purpose, the following assumptions on the given system are required: (i) (A, B) is stabilizable; (ii) (A, C_1) is detectable; and (iii) (A, B, C_2, D_2) is right invertible and has no invariant zeros at $s = 0$. It is well understood in the literature that these assumptions are standard and necessary.

The paper is organized as follows: Section 2 deals with the theory of the CNF control for the state feedback case, whereas Section 3 deals with the detailed development of the CNF design with the full order measurement feedback and the reduced order measurement cases. We will address the issue on the selection of some key design parameters in Section 4. The proposed technique will then be illustrated by a numerical example in Section 5. Some concluding remarks will be drawn in Section 6.

2. The state feedback case

We first proceed to develop a composite nonlinear feedback control technique for the case when all the state variables of the plant Σ of (1) are measurable, i.e., $y = x$. The design will be done in three steps, which is a natural extension of the results of Chen et al. [3]. We have the following step-by-step design procedure.

Step S1: Design a linear feedback law

$$u_{\mathrm{L}} = Fx + Gr, \tag{3}$$

where $r \in \mathbb{R}^l$ contains a set of step references. The state feedback gain matrix $F \in \mathbb{R}^{m\times n}$ is chosen such that the closed-loop system matrix $A + BF$ is asymptotically stable and the resulting closed-loop system transfer matrix, i.e., $D_2 + (C_2 + D_2F)(sI - A - BF)^{-1}B$, has certain desired properties, e.g., having a small dominating damping ratio in each channel. We note that such an F can be worked out using some well-studied methods such as the LQR, H_∞ and H_2 optimization approaches (see, e.g., [1,2,11]). Furthermore, G is an $m \times l$ constant matrix and is given by

$$G := G_0'(G_0G_0')^{-1} \tag{4}$$

with $G_0 := D_2 - (C_2 + D_2F)(A + BF)^{-1}B$. Here we note that both G_0 and G are well defined because $A + BF$ is stable, and (A, B, C_2, D_2) is right invertible and has no invariant zeros at $s = 0$, which implies