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Remote control of LTI systems over networks with state quantization☆

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Abstract

We consider a remote control system, where the plant and the controller are connected by a network cable, and study the problem of designing quantizers for its stabilization. It is assumed that the computation available on the plant side in the sensor/actuator is limited and also that broadcast of messages is allowed over the channel.

Keywords: Data rate limitation; Quantization; Remote control; Sampled-data systems; Stabilizability

1. Introduction

Quantization effects are present in most control systems, as they heavily rely on digital components, and have long been studied. Recently, such effects that arise in systems with networks have received considerable attention. When a digital network is present in a feedback system, quantization levels determine the data rate for the transmission of control-related signals and hence the cost for communication. While quantization levels may not be the largest factor contributing to high data rates [8], careful design of communication is mandatory in large-scale systems, which certainly involves coding aspects.

This viewpoint has prompted studies on design methods for quantizers efficient in terms of data rate. In particular, stabilization problems for LTI systems with one quantizer are considered in, e.g., [1–3,5–7,9–14]. In these problems, the communication over the channel is in one direction, and signals are sent from the sensor side directly to the actuator side. Hence, only one quantizer, or coder, is present. The minimum data rates are derived for such problems for deterministic cases in [3,9,12,13] and for stochastic cases in [10,11]; these minimum rates can be achieved through the use of quantizers that are time varying and require memory on both the coders and the decoders.

In this paper, we study quantization issues in a remote control system, as depicted in Fig. 1. This is another realistic setup in the context of control over networks. Here, the plant and the controller are physically

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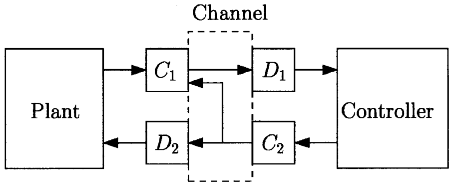

Fig. 1. Remote control system with coders C_i and decoders D_i, $i = 1, 2$.

distant, and the sensor and the actuator are connected to the controller over a shared network. The sensor output is encoded in the coder C_1, transmitted over the channel, and then decoded at the decoder D_1 on the controller side. Similarly, the controller output is sent over the channel to the actuator through the coder C_2 and the decoder D_2. The setup is more complicated than the one mentioned above, and clearly there are two quantizers.

Here, our focus is on problems related to quantization, and thus we ignore the issue of time delay. This should not be construed, however, as saying that time delay issues are not important in remote control applications. They could be important contributing factors to performance degradation in some applications (see, e.g., [8]), but within the scope of this paper, to maintain the focus on quantization, we assume that delay effects are negligible. We note that this assumption may be justified, e.g., when remote control takes place in a short distance where no delay due to queuing in routers is present. For example, in an automobile engine, the controller is separately located to avoid heat and vibration.

Specifically, we develop a design method for the two quantizers to achieve stabilization of the overall system and to use the data rate efficiently. We employ time-invariant, memoryless quantizers and, in particular, those of the logarithmic type. In [2], such quantizers were first considered and were shown to be the “coarsest” type under a certain optimization problem for stabilization of discrete-time systems. Logarithmic quantizers were also studied in a sampled-data setup in [6], where the emphasis was on quadratic stability in the continuous-time domain. Here too, logarithmic quantizers were shown to be more efficient over other quantizers such as the uniform ones. In this paper, we follow this approach of [6] for the remote control problem, which involves its extension to a state quantization problem.

One of the objectives of the study is to characterize the tradeoffs in communication arising in this setup. One is between the rates in the two directions of communication (to and from the controller). We find that, to maintain stability, a lower rate in one direction may result in a higher rate in the other. Another tradeoff is between the data rate and the performance with respect to the degree of stability. That is, more data rate implies that the state of the system decays faster, and vice versa.

In the problem setup, we incorporate several features of remote control. It is natural that the computation available in the coder and the decoder on the plant side is very limited, and thus most computation takes place in the controller. This aspect is modeled by the use of time-invariant, memoryless quantizers. We note however that minimum data rates as in [3,9,12,13] seem difficult to obtain for this class of quantizers. This is mainly because the designs usually rely on the Lyapunov methods, as we shall see in our development; see [13] for more discussion. We also make use of the broadcast feature in the network, which allows multiple nodes to receive a single message at the same time without extra data rate. We examine the case where the controller transmits messages to both C_1 and D_2 as in Fig. 1.

This paper is organized as follows. In Section 2, we give the definition of quantizers used throughout the paper. In Section 3, we state our problem of remote control over networks. This problem motivates us to study in Section 4 a state quantization problem formulated as an extension of the quantization methods in [6]. The results will be applied to solving the remote control problem in Section 5.