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Tracking control: Performance funnels and prescribed transient behaviour

Achim Ilchmann[a,*], Eugene P. Ryan[b], Stephan Trenn[a]

[a] *Institute of Mathematics, Technical University Ilmenau, Weimarer Straße 25, 98693 Ilmenau, Germany*
[b] *Department of Mathematical Sciences, University of Bath, Claverton Down, Bath BA2 7AY, UK*

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Abstract

Tracking of a reference signal (assumed bounded with essentially bounded derivative) is considered in the context of a class of nonlinear systems, with output y, described by functional differential equations (a generalization of the class of linear minimum-phase systems of relative degree one with positive high-frequency gain). The primary control objective is tracking with prescribed accuracy: given $\lambda > 0$ (arbitrarily small), determine a feedback strategy which ensures that for every admissible system and reference signal, the tracking error $e = y - r$ is ultimately smaller than λ (that is, $\|e(t)\| < \lambda$ for all t sufficiently large). The second objective is guaranteed transient performance: the evolution of the tracking error should be contained in a prescribed performance funnel $\mathscr{F}$. Adopting the simple non-adaptive feedback control structure $u(t) = -k(t)e(t)$, it is shown that the above objectives can be attained if the gain is generated by the nonlinear, memoryless feedback $k(t) = K_{\mathscr{F}}(t, e(t))$, where $K_{\mathscr{F}}$ is any continuous function exhibiting two specific properties, the first of which ensures that if $(t, e(t))$ approaches the funnel boundary, then the gain attains values sufficiently large to preclude boundary contact, and the second of which obviates the need for large gain values away from the funnel boundary.

Keywords: Output feedback; Transient behaviour; Tracking; Functional differential equations

1. Introduction

By way of motivation, consider the well-studied (see, for example, [4,6,9]) class of finite-dimensional, real, linear, minimum-phase, M-input ($u(t)$), M-output ($y(t)$) systems of relative degree one having high-frequency gain $B \in \mathbb{R}^{M\times M}$ with $B + B^{\mathrm{T}} > 0$. Systems of this class can, in suitable coordinates, be expressed in the form of two

* Corresponding author.
E-mail addresses: achim.ilchmann@tu-ilmenau.de (A. Ilchmann), epr@maths.bath.ac.uk (E.P. Ryan), stephan.trenn@stud.tu-ilmenau.de (S. Trenn).

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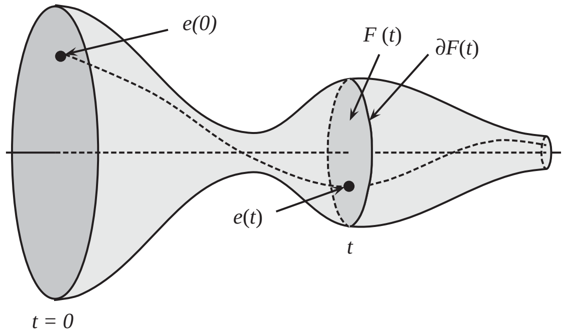

Fig. 1. Performance funnel $\mathscr{F}$.

coupled subsystems

$$\left.\begin{array}{ll}\dot{y}(t) = A_1 y(t) + A_2 z(t) + Bu(t), & y(0) = y^0, \\ \dot{z}(t) = A_3 y(t) + A_4 z(t), & z(0) = z^0,\end{array}\right\} \tag{1}$$

with $y(t), u(t) \in \mathbb{R}^M$, $z(t) \in \mathbb{R}^{N-M}$, and where A_4 has its spectrum in the open left half complex plane. Introducing the linear operator T given by

$$(Ty)(t) := A_1 y(t) + A_2 \int_0^t \exp(A_4(t-s)) A_3 y(s)\, \mathrm{d}s \tag{2}$$

and the function p given by $p(t) := A_2 \exp(A_4 t) z^0$, system (1) can be interpreted as

$$\dot{y}(t) = p(t) + (Ty)(t) + Bu(t), \quad y(0) = y^0. \tag{3}$$

In a precursor [2] to the present paper, (1) formed a prototype subclass of a considerably more general class of nonlinear systems described by functional differential equations of the form

$$\dot{y}(t) = f(p(t), (Ty)(t), u(t)), \quad y_{[-h,0]} = y^0,$$

where, loosely speaking, the parameter $h \geqslant 0$ quantifies system "memory", p may be thought of as a (bounded) disturbance term, and T is a nonlinear causal operator. Whilst a full description of the system class is postponed to Section 2, we remark here that diverse phenomena are incorporated within the class including, for example, diffusion processes, delays (both point and distributed) and hysteretic effects. For this general system class, the problem of output tracking with prescribed asymptotic accuracy and prescribed transient output behaviour was formulated in [2] in terms of a performance funnel $\mathscr{F}$ determined by the graph of the set-valued map $t \mapsto F(t) = \{e | \varphi(t)\|e\| < 1\} \subset \mathbb{R}^M$ for suitably chosen φ; the goal was a control structure which, for every admissible system and reference signal, ensures that the graph of the tracking error $e(\cdot)$ is contained in $\mathscr{F}$ (Fig. 1). This goal was achieved by the simple control structure $u(t) = -k(t)e(t)$ with the gain generated by a nonlinear, memoryless feedback law of the form $k(t) = K_{\mathscr{F}}(t, e(t))$, where $K_{\mathscr{F}}$ is a continuous function such that, loosely speaking, the reciprocal $1/K_{\mathscr{F}}(t, e)$ provides a particular measure of distance of (t, e) from the boundary $\partial\mathscr{F}$ of the funnel $\mathscr{F}$ (with the effect that if the error approaches the boundary, then the gain increases which, in conjunction with a high-gain property of the underlying system class, precludes contact with the boundary) (Fig. 2). The purpose of the present paper, vis-a-vis its precursor [2], is to extend the choice of admissible gain functions $K_{\mathscr{F}}$, allowing for greater flexibility in the choice of measure of the distance to the funnel boundary. Colloquially speaking, the controllers in [2] look "vertically" in the funnel in the sense that, at time t, only the instantaneous funnel information $F(t)$ is used. This approach