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Short communication

A quantum-behaved evolutionary algorithm based on the Bloch spherical search

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ABSTRACT

In order to enhance the optimization ability of the quantum evolutionary algorithms, a new quantum-behaved evolutionary algorithm is proposed. In this algorithm, the search mechanism is established based on the Bloch sphere. First, the individuals are expressed by qubits described on the Bloch sphere, then the rotation axis is established by Pauli matrixes, and the evolution search is realized by rotating qubits on the Bloch sphere about the rotating axis. In order to avoid premature convergence, the mutation of individuals is achieved by the *Hadamard* gates. Such rotation can make the current qubit approximate the target qubit along with the great circle on the Bloch sphere, which can accelerate optimization process. Taking the function extreme value optimization as an example, the experimental results show that the proposed algorithm is obviously superior to other similar algorithms.

1. Introduction

Quantum computing is an emerging interdisciplinary based on combination of quantum mechanics and information science, and its integration with the intelligent optimization algorithm began in the 1990s. In 1996, Ajit et al. proposed quantum-inspired genetic algorithms [1], and successfully solved the TSP problem, creating a new direction of integrating quantum computation and evolutionary computation. From the algorithm principle perspective, it is very similar to a isolated niche genetic algorithm, and the quantum behavior is not obvious. In 2000, Han et al. proposed a genetic quantum algorithm [2], in which, the qubit's state vectors are introduced genetic coding, the quantum rotation gates are used to achieve the adjustment of chromosomal genes, and a gene adjusting strategy is presented. In 2001, Han et al. proposed a parallel quantum genetic algorithm to solve combinatorial optimization problems [3]. By means of the probability properties of quantum state, the quantum chromosome can represent a superposition state of the multiple approximate solutions. In 2002, based on the Ref. [3], Han et al. proposed a quantum-inspired evolutionary algorithm [4], compared with the traditional evolutionary algorithm, its advantage is a better ability to maintain the population diversity. In 2004, Hichem et al. presented a new algorithm for solving the traveling salesman problems [5], which extended the standard genetic algorithm by combining them to some concepts and principles provided from quantum computing field such as qubits, states superposition and interference. Since then, many quantum evolutionary algorithms may be regarded as the improvements to some extent based on the above algorithms [6–9], and these algorithms are mainly applied to combinatorial optimization. By investigating the various quantum intelligent optimization algorithms existed at present, the binary coding based on qubits measure is primary encoding method and the qubit phase changing based on quantum rotation gates is primary evolution method. Firstly, the process of binary encoding obtained by measuring the state of qubits on chromosome is a

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probability operation with a great randomicity and blindness and so, the individual chromosome is possible to degenerate as colony chromosomes evolve. Secondly, although binary coding is suitable for certain optimization such as the 0–1 knapsack problem, traveling salesman problem and so on, it is not suitable for numerical value optimization such as function extremum optimization and neural network weights optimization, because many computations are made from frequent encoding and decoding. Thirdly, the direction of rotation angle of quantum rotation gates must be determined in the optimization process. At present, the direction of rotation angle is usually determined by a query table, proposed in Ref. [2] by Han, that is inefficient on account of to dealing with many conditional judgments. Fourthly, in quantum mechanics, the qubits are usually described by vectors on the Bloch sphere. However, the qubits in current algorithm are described by vectors in the unit circle, which the quantum characteristics are weakened because the only a phase parameter is contained. In 2008, we proposed a quantum-inspired evolutionary algorithm for continuous space optimization based on Bloch coordinates of qubits [10]. This algorithm adopted the qubits' Bloch spherical coordinate coding, had two adjustable parameters, and shew some good optimization performances. However, in this algorithm, the adjustment matching for two parameters was not resolved, which affected the optimization ability to further improve. Based on the above problems, we propose a quantum-behaved evolutionary algorithm (QBEA) based on the Bloch spherical search. Unlike traditional quantum intelligent optimization algorithm, in the proposed algorithm, this algorithm uses Pauli matrices to establish the rotation axis, uses qubit's pivoting to achieve evolutionary search, and uses the *Hadamard* gates to achieve mutation. This evolutionary approach can simultaneously adjust the two parameters of a qubit, and can automatically achieve the best matching between the two adjustments. With the typical function extremum optimization, and comparison with other algorithms, the experimental results verify the effectiveness of the QBEA.

2. The principle of QBEA

2.1. The spherical description of qubits

In quantum computing, a qubit is a two-level quantum system, described by a two-dimensional complex Hilbert space. From the superposition principles, any state of the qubit may be written as

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle, \tag{1}$$

where $0 \leqslant \theta \leqslant \pi$, $0 \leqslant \phi \leqslant 2\pi$.

Therefore, unlike the classical bit, which can only be set equal to 0 or 1, the qubit resides in a vector space parametrized by the continuous variables θ and ϕ. Thus, a continum of states is allowed. The Bloch sphere representation is useful in thinking about qubits since it provides a geometric picture of the qubit and of the transformations that one can operate on the state of a qubit. Owing to the normalization condition, the qubit's state can be represented by a point on a sphere of unit radius, called the Bloch sphere. This sphere can be embedded in a three-dimensional space of Cartesian coordinates ($x = \cos\phi\sin\theta, y = \sin\phi\sin\theta, z = \cos\theta$). Thus the state $|\varphi\rangle$ can be written as

$$|\varphi\rangle = \left[\sqrt{\frac{1+z}{2}}, \frac{x+iy}{\sqrt{2(1+z)}}\right]^{\mathrm{T}}. \tag{2}$$

By definition, a Bloch vector is a vector whose components (x, y, z) single out a point on the Bloch shpere. We can say that the angles θ and ϕ define a Bloch vector, as shown in Fig. 1, where the points corresponding to the following states are shown: $|A\rangle = [1,0]^{\mathrm{T}}$, $|B\rangle = [0,1]^{\mathrm{T}}$, $|C\rangle = |E\rangle = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]^{\mathrm{T}}$, $|D\rangle = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]^{\mathrm{T}}$, $|F\rangle = \left[\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right]^{\mathrm{T}}$, $|G\rangle = \left[\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right]^{\mathrm{T}}$.

2.2. The encoding method of BQEA

In QBEA, all individuals are encoded by qubits described on the Bloch sphere. Set the colony size equal to m, and the space dimension equal to n. Then the ith individual is encoded as

$$p_i = [|\varphi_{i,1}\rangle, |\varphi_{i,2}\rangle, \cdots, |\varphi_{i,n}\rangle], \tag{3}$$

where $|\varphi_{i,j}\rangle = \left[\cos\frac{\theta_{i,j}}{2}, e^{i\phi}\sin\frac{\theta_{i,j}}{2}\right]^{\mathrm{T}}$; $0 \leqslant \theta_{i,j} \leqslant \pi$; $0 \leqslant \phi_{i,j} \leqslant 2\pi$; $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

As the optimization is performed in $[-1, 1]^n$, which has nothing to do with the specific issues, hence, the proposed method has good adaptability for a variety of optimization problems.

2.3. Projective measurement of qubits

From the principles of quantum computing, the coordinates x, y, and z of a qubit on the Bloch sphere can be measured by such the following Pauli operators written in the computational basis as