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Short communication

Global bifurcations of a taut string with 1:2 internal resonance ☆

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ABSTRACT

The global bifurcations of a taut string are investigated with the case of 1:2 internal resonance. The method of multiple scales is applied to obtain a system of autonomous ordinary differential equations. Based on the normal form theory, the desired form for the global perturbation method is obtained. Then the method developed by Kovacic and Wiggins is used to find explicit sufficient conditions for chaos to occur by identifying the existence of a Silnikov-type homoclinic orbit. Finally, numerical results obtained by using fourth-order Runge–Kutta method agree with the theoretical analysis at least qualitatively.

1. Introduction

Taut string is a basic mechanical element. The research on taut string has a long history. As early as the eighteen century, Euler, Bernoulli and Lagrange [1] have done their best to study the vibrations of taut strings. Some mechanical structures can be simplified as a taut strings. For example, inclined cable and cable-stayed bridge. Lilien [2] simplified an inclined cable as a taut string, assumed the axial displacement function was the cosine function and obtained the motion equations and analyzed the resonance regions. Nayfeh et al. [3] analyzed the nonlinear response of a taut string to an end excitation with components both parallel and transverse to its axis. Royer-Carfagni [4] simplified the inclined cable as a taut string and studied the response of the taut string excited by a sine function. Gattulli et al. [5] studied the modal localization of cable-stayed bridges on basis of the hypothesis of vanishing cable sag (i.e. the taut string). Ren et al. [6] studied the dynamic characteristics of a large span cable-stayed bridge with both experimental and analytical method by assuming cable as a taut string. Moreover, O'Reilly [7] has investigated the global bifurcations of a damped elastic string which allow the string to change its direction of whirling.

Several methods have been developed to study the global bifurcation behaviors in high-dimensional nonlinear systems that possess homoclinic or heteroclinic orbits. The methods include Melnikov method, global perturbation method and the energy-phase method. Wiggins [8] divided four-dimensional perturbed Hamiltonian systems into three types and investigated the global bifurcation behaviors for the three basic systems through Melnikov method. Later on, global perturbation technique [9] proposed by Kovacic and Wiggins can be used to detect homoclinic and heteroclinic orbits for perturbed resonant system. Based on the technique, the global dynamics of the various mechanic models [10–18] have been studied. The energy-phase method [19–21], which is developed by Haller and Wiggins, is used to investigate the existence of Silnikov type single-pulse homoclinic orbits and multipulse homoclinic orbits for the both cases of pure Hamiltonian perturbation

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and dissipative perturbation. Applying this method, Malhotra et al. [22] investigated the existence of multipulse orbits in the motion of flexible spinning discs; Yu et al. [23] detected the presence of multipulse orbits in externally excited cyclic systems for the resonant case in both Hamiltonian and dissipative perturbations cases; Chen et al. [24] studied the chaotic dynamics in suspended cables with 1:1 internal resonance. Note that Zhang et al.[16] studied the Silnikov type single pulse homoclinic orbit of a cable with 1:1 internal resonance and Chen et al. [17] investigated the Silnikov type single pulse homoclinic orbit of an inclined cable with 1:1 internal resonance. Later on, Chen et al. [24] investigated the Silnikov type multipulse homoclinic orbits of suspend cables with 1:1 internal resonance. In this paper, the Silnikov type single pulse homoclinic orbit of a taut string with 1:2 internal resonance will be considered by means of normal form theory and global perturbation method.

The work is organized as follows: in Section 2, the equations of the motion in a desired form are determined in aid of the method of multiple scales and the normal form theory. The dynamics of unperturbed system is analyzed in Section 3, which is followed in Section 4 by the details of the dynamics on perturbed system. In Section 5, numerical simulations are given to confirm the analysis predictions and the work ends in Section 6 with a conclusion of all the results.

2. Formulation of the problem

We consider the following two-degree-of-freedom system of a taut string,

$$x'' + 2\zeta_1\omega_1 x' + \omega_1^2(1 + e_0\cos(\omega t))x + x(\gamma_1 x^2 + \gamma_2 y^2) = 0, \tag{1a}$$

$$y'' + 2\zeta_2\omega_2 y' + \omega_2^2(1 + e_0\cos(\omega t))y + y(\gamma_1 x^2 + \gamma_2 y^2) = -\delta e_0\cos(\omega t), \tag{1b}$$

where x and y are the two displacements. The prime denotes differentiation with respect to the time t. In order to obtain a system which is suitable to use the method of multiple scales, the system (1) can be transformed as follows,

$$x'' + \omega_1^2 x = -\varepsilon(2\mu_1 x' + \omega_1^2 x e_0\cos\omega t + \gamma_1 x^3 + \gamma_2 x y^2), \tag{2a}$$

$$y'' + \omega_2^2 y = -\varepsilon(2\mu_2 y' + \omega_2^2 y e_0\cos\omega t + \gamma_1 x^2 y + \gamma_2 y^3 + \delta e_0\cos\omega t), \tag{2b}$$

where $e_0 \to \varepsilon e_0,\ \zeta_i\omega_i \to \varepsilon\mu_i(i = 1, 2),\ \gamma_i \to \varepsilon\gamma_i(i = 1, 2)$. The resonant relations are $\omega_1^2 = \frac{\omega^2}{4} + \varepsilon\sigma_1, \omega_2^2 = \omega^2 + \varepsilon\sigma_2$, where $\sigma_i(i = 1, 2)$ are two detuning parameters and ε is a small perturbation parameter. Without considering the mechanical meaning, we let $\omega = 1$. Now assuming that system (2) have an approximate solution in the form

$$\begin{aligned} x(t) &= x^0(T_0, T_1) + \varepsilon x^1(T_0, T_1) + \cdots, \\ y(t) &= y^0(T_0, T_1) + \varepsilon y^1(T_0, T_1) + \cdots, \end{aligned} \tag{3}$$

where $T_i = \varepsilon^i t(i = 0, 1)$. Substituting (3) into (2)**?** and equating coefficients of like powers of ε yields,

$O(1)$:

$$D_0^2 x^0 + \frac{\omega^2}{4}x^0 = 0, \quad D_0^2 y^0 + \omega^2 y^0 = 0. \tag{4}$$

$O(\varepsilon)$:

$$\begin{aligned} D_0^2 x^1 + \frac{\omega^2}{4}x^1 &= -2D_0D_1x^0 - 2\mu_1 D_0 x^0 - \sigma_1 x^0 - \omega^2 x^0 e_0\cos\omega t - \gamma_1(x^0)^3 - \gamma_2 x^0(y^0)^2, \\ D_0^2 y^1 + \omega^2 y^1 &= -2D_0D_1y^0 - 2\mu_2 D_0 y^0 - \sigma_2 y^0 - \omega^2 y^0 e_0\cos\omega t - \gamma_1(x^0)^2 y - \gamma_2(y^0)^3 - \delta e_0 cos\omega T_0, \end{aligned} \tag{5}$$

where $D_k = \partial/\partial T_k(k = 0, 1)$.

The general solution of Eq. (4) can be written as follows,

$$x^0 = A_1(T_1)e^{\frac{i\omega T_0}{2}} + \bar{A}_1(T_1)e^{\frac{-i\omega T_0}{2}}, \quad y^0 = A_2(T_1)e^{i\omega T_0} + \bar{A}_2(T_1)e^{-i\omega T_0}, \tag{6}$$

where $\bar{A}_i(T_1)$ $(i = 1, 2)$ denote the complex conjugate of $A_i(T_1)$ $(i = 1, 2)$ and the functions $A_i(T_1)$ $(i = 1, 2)$ are to be determined by satisfying the solvability conditions. Substituting (6) into (5), the solvability conditions are obtained as follows:

$$-i(\dot{A}_1 + \mu_1 A_1) - \sigma_1 A_1 - \frac{1}{2}e_0\bar{A}_1 - 3\gamma_1 A_1^2\bar{A}_1 - 2\gamma_2 A_1 A_2\bar{A}_2 = 0, \tag{7a}$$

$$-2i(\dot{A}_2 + \mu_2 A_2) - \sigma_2 A_2 - 3\gamma_2 A_2^2\bar{A}_2 - 2\gamma_1 A_1\bar{A}_1 A_2 - \frac{\delta e_0}{2} = 0, \tag{7b}$$

where a prime denotes differentiation with respect to the time T_1. Now letting

$$A_1(T_1) = x_1(T_1) + iy_1(T_1), \quad A_2(T_1) = x_2(T_1) + iy_2(T_1). \tag{8}$$

Substituting (8) into (7) and separating the result into real and imaginary parts yields