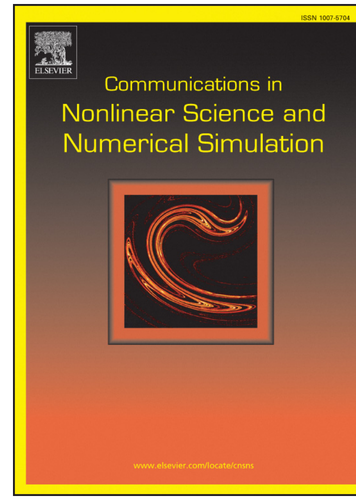

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Escape beam statistics and dynamical properties for a periodically corrugated waveguide

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Abstract

Some escape and dynamical properties for a beam of light inside a corrugated waveguide are discussed by using Fresnel reflectance. The system is described by a mapping and is controlled by a parameter δ defining a transition from integrability ($\delta = 0$) to non integrability ($\delta \neq 0$). The phase space is mixed containing periodic islands, chaotic seas and invariant tori. The histogram of escaping orbits is shown to be scaling invariant with respect to δ. The waveguide is immersed in a region with different refractive index. Different optical materials are used to overcame the results.

Key words: Corrugated waveguide, Fresnel, escape properties

1. Introduction

The Fresnel equations were firstly obtained in the early part of the 19th century and describe the behaviour of light when moving between media with different refractive indexes. The reflection of light that the equations predict is known as Fresnel reflection. When light moves from medium interior of a waveguide with a given refractive index n_1 towards a boundary surrounding a medium with refractive index n_2 ($n_1 > n_2$), both reflection and refraction of the beam light may occur. The Fresnel equations describe what fraction of the light is reflected (part of the beam escapes and it is refracted) when the incident angle θ_i (between incident beam and normal vector to boundary at incident point) is smaller than a critical angle given by

$$A_C = \arcsin(n_{21}), \tag{1}$$

where $n_{21} = n_2/n_1$. A total reflection occurs and the beam stays totally into the waveguide for incident angle larger than the critical angle. The fraction of the incident power is given by the reflectance R and it is equal to the unity for the case $\theta_i > A_C$. However for $\theta_i \leq A_C$ the calculations of R depend on polarization of the incident ray. Two cases may be analysed: (i) The incident light is p-polarized, where the electric field is in the incidence plane which contains the incident beam and the normal vector at the surface in the incident point; (ii) The incident light is s-polarized, where the electric field is in a perpendicular direction to p-polarized. For a s-polarized ray of light, the reflection coefficient is given by

$$R_s = \left(\frac{\cos(\theta_i) - n_{21}\sqrt{1 - \left(\frac{\sin(\theta_i)}{n_{21}}\right)^2}}{\cos(\theta_i) + n_{21}\sqrt{1 - \left(\frac{\sin(\theta_i)}{n_{21}}\right)^2}} \right)^2 , \tag{2}$$

and for a p-polarized light, the reflection coefficient is equal to

$$R_p = \left(\frac{-n_{21}\cos(\theta_i) + \sqrt{1 - \left(\frac{\sin(\theta_i)}{n_{21}}\right)^2}}{n_{21}\cos(\theta_i) + \sqrt{1 - \left(\frac{\sin(\theta_i)}{n_{21}}\right)^2}} \right)^2 . \tag{3}$$

These equations can be used to another polarization ray of incident light by decomposition to s-polarization and p-polarization. On the other hand when the beam is unpolarized (i.e. rays decomposition to these polarizations have null mean) the beam may be considered containing an equal mix of these polarizations rays. In this case, we can see by Fig. 1(a) that after the incidence, with angle less than the critical angle, that the beam becomes predominantly s-polarized (partial polarization). The beam can be totally s-polarized if the incident angle is equal to the Brewster angle ($A_B = \arctan(n_{21})$ i.e.. $\sin(A_B)/\cos(A_B) = n_{21}$) [1, 2, 3], where R_p is null.

The problem of a beam escaping from a waveguide with chaotic ray dynamics was considered earlier by Zaslavsky and Abdullaev in Ref. [4] while the phase space structure of escaping particles was later considered by Makarov, Uleysky and Prandts in Ref. [5]. Applications of the formalism using waveguides can be seen in the determination of the band structure and quantum Poincaré sections of a classically chaotic quantum rippled channel [6], classical versus quantum structure of the scattering probability matrix [7] deterministic scattering of a quantum particle in a two-dimensional ballistic waveguide [8], creation of slow waves near cut-off frequencies [9] and squeezed-light generation in nonlinear planar waveguide [10]. Another application considers transport through a finite $GaAs/Al_xGa_{1-x}As$ hetero-structure [11], quantised ballistic conductance in a periodically modulated quantum channel [12], quantum transport in ballistic cavities [13], clas-

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