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Emergence of density dynamics by surface interpolation in elementary cellular automata

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ABSTRACT

A classic problem in elementary cellular automata (ECAs) is the specification of numerical tools to represent and study their dynamical behaviour. Mean field theory and basins of attraction have been commonly used; however, although the first case gives the long term estimation of density, frequently it does not show an adequate approximation for the step-by-step temporal behaviour; mainly for non-trivial behaviour. In the second case, basins of attraction display a complete representation of the evolution of an ECA, but they are limited up to configurations of 32 cells; and for the same ECA, one can obtain tens of basins to analyse. This paper is devoted to represent the dynamics of density in ECAs for hundreds of cells using only two surfaces calculated by the nearest-neighbour interpolation. A diversity of surfaces emerges in this analysis. Consequently, we propose a surface and histogram based classification for periodic, chaotic and complex ECA.

1. Introduction

Elementary cellular automata (ECAs) have been widely studied because they are able to produce interesting dynamical behaviour based on very simple local interactions [1,2].

A classic problem in ECAs has been the specification of numerical tools and methods to analyse, predict and classify their dynamical behaviour.

The use of mean field theory for this task was initiated by Wolfram in [3]. Then, it was developed by Gutowitz [4], McIntosh [5], and Martinez [6] among many others. The idea is to model the long-term behaviour of density with a polynomial specified by the evolution rule of an ECA.

These polynomials are commonly nonlinear and are directly related to the dynamics of ECAs.

Another way to represent the dynamics of ECAs is using basins of attraction [7]. In this case, we have a graphic representation of all evolution in an ECA, and the branches and basins in these graphs describe the dynamics of the system. However, this technique can produce several basins for the same automaton and it is limited up to 32 cells because of the exponential growth.

On the other hand, recent papers have exposed numerical techniques for detecting chaotic behaviour in ECAs. These studies take a sample of evolutions to estimate Lyapunov exponents [8, 9, 10], response curves [11] and Fourier spectra [12]. In particular, ECAs have been analysed in [13] using a Walsh transformation in order to know their efficiency in the generation

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of random sequences. A relationship between conserved quantities (density classification and nonequilibrium phase transition) and the dynamics of ECAs is presented in [14]; and empirical measures for the intermediate conservation degree of properties in ECAs have been proposed in [15].

With these results as background, the aim of this paper is to obtain a representation for the dynamical behaviour of density in ECAs by surface interpolation. We take the density and the binary value of configurations and their evolutions in an ECA, and with them we calculate two surfaces describing the dynamic behaviour of these values using the nearest-neighbour interpolation.

For a small number of cells (22 cells), all the configurations are taken. Thus, the surfaces obtained represent in a complete way the dynamics of density in time. Further, for a greater number of cells (100, 200 and 400), the surfaces are estimated with a sample of random configurations. In all these cases, we have analysed ECAs with periodic boundary conditions.

This process represents the dynamical behaviour of density in ECAs with two surfaces. Besides, these surfaces are analysed to detect periods in time and classify periodic, chaotic and complex behaviour in ECAs; obtaining interesting results.

The paper is organised as follows. Section 2 presents the definition of ECAs and shows the problems of analysing their dynamics with mean field theory and basins of attraction. Section 3 describes the process to obtain surfaces that describe the dynamical behaviour of density. It shows as well how these surfaces are studied to understand and classify the dynamics of ECAs. Section 4 explains how these surfaces are estimated for a greater number of cells taking a sample of evolutions; proposing a surface and histogram based classification of ECAs. Section 5 applies this classification to the 88 ECA representative rules. The final sections provide the discussion and the conclusions of the paper.

2. Preliminars

2.1. Elementary cellular automata

This paper is concerned with the analysis of elementary cellular automata with periodic boundary conditions in the terminology presented by [16,17].

An elementary cellular automaton (ECA) is defined by a finite array of locally connected cells x_i where $i \in \{0, \ldots, n-1\}$ (ring of the integers modulo n) and each x takes a value from a binary set of states $\Sigma = \{0, 1\}$. A sequence of cells $\{x_i\}$ of finite length n is a configuration c of the ECA. The set of finite configurations is represented as Σ^n. An evolution is described by a sequence of configurations $\{c_i\}$ given by the mapping $\Phi : \Sigma^n \rightarrow \Sigma^n$ such that:

$$\Phi(c^t) \rightarrow c^{t+1} \quad (1)$$

where t indicates time and the sequence of cell states in c defines the global state. The cell states in c^t are updated simultaneously by the same evolution rule φ in the following way:

$$\varphi(x^t_{i-1}, x^t_i, x^t_{i+1}) \rightarrow x^{t+1}_i \quad (2)$$

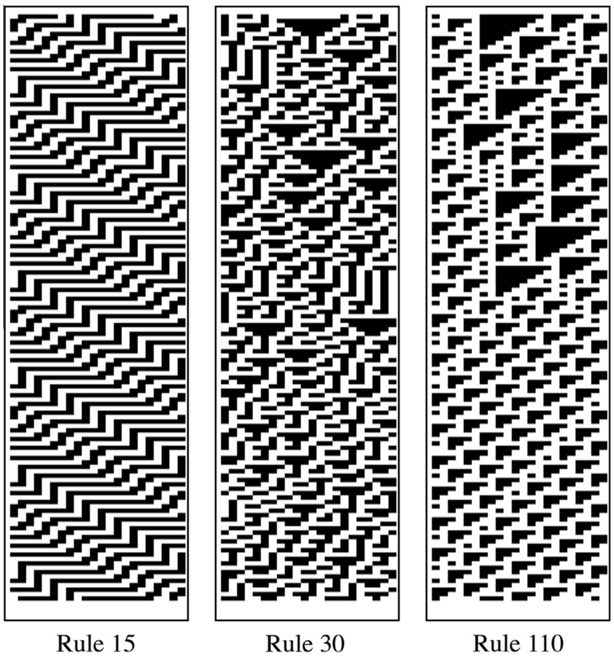

Fig. 1. Examples of evolutions in ECAs. In these evolutions we can see examples of periodic (rule 15), chaotic (rule 30) and complex dynamics (rule 110).