Commun Nonlinear Sci Numer Simulat 19 (2014) 977–990

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Synchronization configurations of two coupled double pendula

Piotr Koluda, Przemyslaw Perlikowski, Krzysztof Czolczynski, Tomasz Kapitaniak *

Division of Dynamics, Technical University of Lodz, Stefanowskiego 1/15, Lodz, Poland

A R T I C L E I N F O

Article history:
Received 1 August 2013
Accepted 9 August 2013
Available online 22 August 2013

Keywords:
Double pendulum
Synchronization
Energy balance

A B S T R A C T

We consider the synchronization of two self-excited double pendula hanging from a horizontal beam which can roll on the parallel surface. We show that such pendula can obtain four different robust synchronous configurations. Our approximate analytical analysis allows to derive the synchronization conditions and explains the observed types of synchronizations. We consider the energy balance in the system and show how the energy is transferred between the pendula via the oscillating beam allowing the pendula' synchronization.

1. Introduction

Groups of oscillators are observed to synchronize in a diverse variety of systems [1,3,15,18,24–26], despite the inevitable differences between the oscillators. Synchronization is commonly the process where two or more systems interact with each other and come to oscillate together. The history of synchronization goes back to the 17th century when Ch. Huygens observed weak synchronization of two pendulum clocks [9]. Recently the phenomenon of the synchronization of the clocks hanging on a common movable beam has been the subject of research by a number of authors [2,4–8,10–14,16,17,19–21]. These studies have explained the phenomenon of synchronization of a number of single pendula. The problem of the synchronization of double pendula is less investigated. Fradkov et al. [22] developed the control system which allows the experimental synchronization of two double pendula. The occurrence of the synchronous rotation of a set of four uncoupled nonidentical double pendula arranged into a cross structure mounted on a vertically excited platform has been studied in [23]. It has been shown that after a transient, many different types of synchronous configurations with the constant phase difference between the pendula can be observed.

In this paper we consider the synchronization of two self-excited double-pendula. The oscillations of each pendulum are self-excited by the escapement mechanism associated with the lower parts (lower pendula) of each double-pendulum. We show that two such double-pendula hanging on the same beam can synchronize both in phase and in antiphase. We give evidence that the observed synchronous states are robust as they exit in the wide range of system parameters and are preserved for the parameters' mismatch (the pendula with different lengths are considered). The performed approximate analytical analysis allows to derive the synchronization conditions and explains the observed types of synchronizations. The energy balance in the system allows to show how the energy is transferred between the pendula via the oscillating beam.

This paper is organized as follows. Section 2 describes the considered model of the coupled double pendula. In Section 3 we derive the energy balance of the synchronized identical pendula. Stable synchronous configurations of double pendula have been identified in Section 4. Section 5 presents the results of our numerical simulations and describes the observed synchronization states. Finally, we summarize our results in Section 6.

* Corresponding author. Tel.: +48 426312231.
E-mail addresses: tomasz.kapitaniak@p.lodz.pl, tomaszka@p.lodz.pl (T. Kapitaniak).

1007-5704/$ - see front matter
http://dx.doi.org/10.1016/j.cnsns.2013.08.008

2. The model

The analyzed system is shown in Fig. 1. It consists of the rigid beam and two double pendula suspended on it. The beam of mass M can move in horizontal direction, its movement is described by coordinate X. The beam is connected to the refuge by a linear spring with stiffness coefficient K_X and linear damper with damping coefficient C_X. Each double pendulum consists of two light beams of lengths L_{ci}, L_{si} and two masses M_{ci} and M_{si}, where $i = 1,2$, mounted at beam's ends. Subscripts s and c describe respectively upper and lower parts (pendula) of each double pendulum (see Fig. 1). The lower parts (pendula) are mounted to the upper parts (pendula) at the distances L_{ai} from the points in which double pendula are mounted to the beam M. The motion of each double-pendulum is described by angles φ_{ci} (lower pendulum) and φ_{si} (upper pendulum). The oscillations of the double pendula are damped by the viscous dampers C_{si} and C_{ci} (not shown in Fig. 1). The lower pendula of each double pendulum are excited by the clock escapement mechanism (described in details in [10]) represented by momentum M_{Di} which provide the energy needed to compensate the energy dissipation due to the viscous friction C_{si}, C_{ci} and to keep the pendula oscillating [1]. This mechanism acts in two successive steps (the first step is followed by the second one and the second one by the first one). In the first step if $0 < (\varphi_{ci} - \varphi_{si}) < \gamma_N$ then $M_{Di} = M_{Ni}$ and when $(\varphi_{ci} - \varphi_{si}) < 0$ or $\gamma_N < (\varphi_{ci} - \varphi_{si})$ then $M_{Di} = 0$, where γ_N and M_{Ni} are constant values which characterize the mechanism. For the second stage one has for $-\gamma_N < (\varphi_{ci} - \varphi_{si}) < 0$ $M_{Di} = -M_{Ni}$ and $M_{Di} = 0$ for $0 < (\varphi_{ci} - \varphi_{si})$ or $-\gamma_N > (\varphi_{ci} - \varphi_{si})$.

Note that the system shown in Fig. 1 can be considered as the two-dimensional model of Huygens' experiment (upper pendula represent clocks' cases and lower pendula clocks' pendula) [10].

The equations of motion of the considered system are as follows:

$$M_{ci}L_{ci}^2\frac{d^2\varphi_{ci}}{dt^2} + M_{ci}L_{ai}L_{ci}\frac{d^2\varphi_{si}}{dt^2}\cos(\varphi_{ci}-\varphi_{si}) + M_{ci}L_{ai}L_{ci}\left(\frac{d\varphi_{si}}{dt}\right)^2\sin(\varphi_{ci}-\varphi_{si}) + M_{ci}L_{ci}\frac{d^2X}{dt^2}\cos\varphi_{ci} + C_{\varphi ci}\left(\frac{d\varphi_{ci}}{dt}-\frac{d\varphi_{si}}{dt}\right) + M_{ci}L_{ci}g\sin\varphi_{ci} = M_{Di}, \quad i=1,2 \tag{1}$$

$$M_{si}L_{si}^2\frac{d^2\varphi_{si}}{dt^2} + M_{ci}L_{ai}^2\frac{d^2\varphi_{si}}{dt^2} + M_{ci}L_{ai}L_{ci}\frac{d^2\varphi_{ci}}{dt^2}\cos(\varphi_{ci}-\varphi_{si}) - M_{ci}L_{ai}L_{ci}\left(\frac{d\varphi_{ci}}{dt}\right)^2\sin(\varphi_{ci}-\varphi_{si}) + M_{si}L_{si}\frac{d^2X}{dt^2}\cos\varphi_{si} + M_{ci}L_{ai}\frac{d^2X}{dt^2}\cos\varphi_{si} + C_{\varphi si}\frac{d\varphi_{si}}{dt} - C_{\varphi ci}\left(\frac{d\varphi_{ci}}{dt}-\frac{d\varphi_{si}}{dt}\right) + M_{si}L_{si}g\sin\varphi_{si} + M_{ci}L_{ai}g\sin\varphi_{si} = -M_{Di}, \tag{2}$$

$$\left(M_B + \sum_{i=1}^{2}(M_{ci}+M_{si})\right)\frac{d^2X}{dt^2} + C_X\frac{dX}{dt} + K_XX = \sum_{i=1}^{2}(M_{si}L_{si}+M_{ci}L_{ai})\left(-\frac{d^2\varphi_{si}}{dt^2}\cos\varphi_{si} + \left(\frac{d\varphi_{si}}{dt}\right)^2\sin\varphi_{si}\right) + \sum_{i=1}^{2}M_{ci}L_{ci}\left(-\frac{d^2\varphi_{ci}}{dt^2}\cos\varphi_{ci} + \left(\frac{d\varphi_{ci}}{dt}\right)^2\sin\varphi_{ci}\right), \tag{3}$$

where $i = 1,2$.

Considering mass M_{c1}, length L_{c1} of the first lower pendulum and gravitational acceleration g as reference parameters one can rewrite Eqs. (1)–(3) in the dimensionless form:

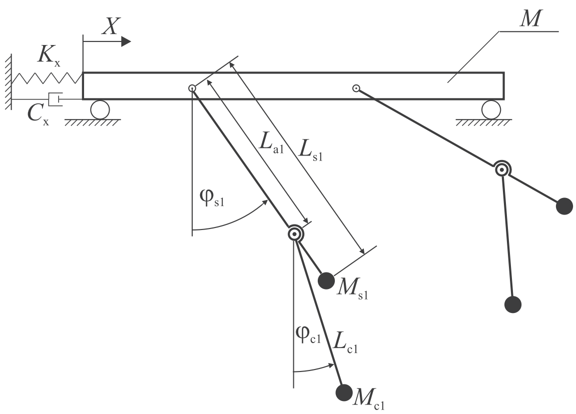

Fig. 1. The model of two double pendula hanging from a horizontal beam.