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Cluster synchronization in colored community network with different order node dynamics

Zhaoyan Wu *

College of Mathematics and Information Science, Jiangxi Normal University, Nanchang 330022, China

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ABSTRACT

In this paper, a colored community network model with different order nodes dynamics is introduced for the first time. The color of nodes or edges are assumed to be identical if they belong to the same community and nonidentical if they belong to different communities. The color of edges between any pair of communities are assumed to be identical if they connect the same pair of communities and nonidentical if they connect different pair of communities. Further, the order of the node dynamics in different communities can be totally different. Firstly, based on the Lyapunov stability theory, adaptive feedback controllers are designed for achieving cluster synchronization. Secondly, periodically intermittent controllers are designed for achieving cluster synchronization and the synchronization conditions are derived by using mathematical induction method and the analysis technique. Finally, several numerical examples are provided to illustrate the effectiveness of the derived theoretical results.

1. Introduction

Recently, many large-scale real systems have been found that they have community structure, such as social and biological networks [1], school friendship networks [2], congressional cosponsorship networks [3], and so on, which can be described by community networks. Generally, in community networks, the connections of the nodes may have higher density if they belong to the same community and lower density if they belong to different communities. Further, the nodes may have the same node dynamics if they are in the same community and different node dynamics if they are in different communities. On the other hand, the interactions between nodes in the same community are usually identical and those in different communities are nonidentical, e.g., the nodes in the first community affect each other only through their first component, while the nodes in the second community affect each other through both the second and third components. Moreover, the interactions between the same pair of communities may be identical and those between different pair of communities may be nonidentical. For example, the second component of nodes in the first community which have connections with nodes in the second community are affected only by the first component of those nodes in the second community, while the second component of nodes in the first community which have connections with nodes in the third community are affected only by the second component of those nodes in the third community. For better describing this kind of community network, referring to colored graph in mathematics, a colored community network model is introduced for the first time. Nodes with different color mean that they have different node dynamics and edges with different color mean that they denote different interactions. Fig. 1 shows a colored community network consisting of 21 nodes with three communities.

* Tel./fax: +86 791 88120360.
E-mail address: zhywu7981@gmail.com

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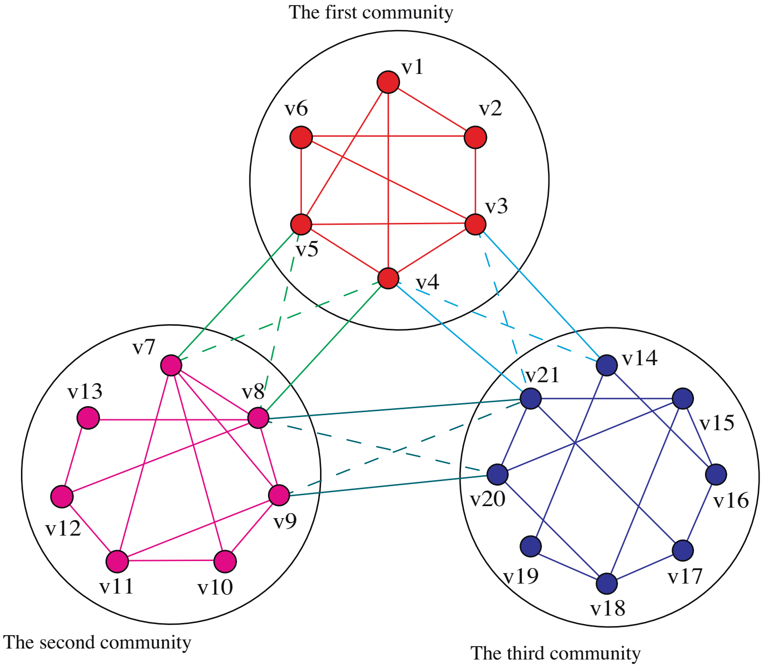

Fig. 1. A colored community network consisting of 21 nodes with three communities.

Synchronization, as a typical collective dynamical behavior of coupled dynamical systems, has been widely studied [4–19]. In community networks, especially those with nonidentical nodes, the nodes belonging to different communities usually tend to synchronize with different states, i.e., the cluster synchronization [20–29]. In [20], cluster synchronization in community networks with nonidentical nodes is studied, and several sufficient conditions for synchronization are obtained analytically. In [25], cluster synchronization in coupled systems with hub structure is investigated. Furthermore, many real complex networks can not synchronize themselves or synchronize with desired orbits. Therefore, proper controllers should be designed to achieve the goals by adopting some control schemes, such as adaptive control [30], feedback control [31], observer-based control [32], impulsive control [28], intermittent control [23,33–37], and so on. Intermittent control scheme, which can be regarded as a transition from continuous-time control to impulsive control, is extensively adopted to design controllers in virtue of the lower control cost and convenient implementation. In [23], cluster synchronization in directed networks is investigated via intermittent control. In [37], exponential synchronization of complex delayed dynamical networks are considered via pinning periodically intermittent control.

Motivated by the above discussions, this paper investigates cluster synchronization of colored community network with different order node dynamics. For achieving the cluster synchronization, adaptive feedback control and intermittent control methods are adopted to design proper controllers. Firstly, according to Lyapunov stability theory, adaptive feedback controllers are designed for achieving the cluster synchronization. Secondly, periodically intermittent controllers are designed for achieving the cluster synchronization and the synchronization conditions are derived by using mathematical induction method and the analysis technique. Noticeably, the outer coupling matrix need not to be symmetrical and irreducible, and the inner matrices need not to be identical.

This paper is organized as follows. Section 2 introduces the colored community network model and some preliminaries. Section 3 considers the cluster synchronization of colored community network via designing adaptive feedback controllers and periodically intermittent controllers, and derives synchronization conditions. Section 4 provides several numerical simulations to verify the correctness and effectiveness of the derived results. Section 5 concludes the paper.

Notation Throughout this paper, for symmetric matrix P, the notation $P > 0 (P < 0)$ means that the matrix P is positive definite (negative definite). I_N denotes the $N \times N$ identity matrix. Q^T denotes the transpose of the matrix Q.

2. Model description and preliminaries

Consider a colored community network consisting of N nodes and p communities with $2 \leqslant p < N$, which can be described by

$$\dot{x}_i(t) = f_k(x_i(t)) + \varepsilon \sum_{j \in V_k, j \neq i} c_{ij} H_{kk}(x_j(t) - x_i(t)) + \varepsilon \sum_{l=1, l \neq k}^{p} \sum_{j \in V_l} c_{ij}(H_{kl} x_j(t) - H_{kk}^{l} x_i(t)), \tag{1}$$