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Nonlinear low frequency water waves in a cylindrical shell subjected to high frequency excitations – Part II: Theoretical analysis

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ABSTRACT

In Part I of this work (Comm. Nonlin. Sci. Numer. Simulat. 18 (2013) 1710–1724), an experimental investigation on nonlinear low-frequency gravity water waves in a cylindrical shell subjected to high-frequency horizontal excitations was reported. To reveal the mechanism of this phenomenon, a theoretical analysis is now presented as Part II of the work. A set of nonlinear equations for two mode interactions is established based on variational principle of fluid-shell coupled system. Theory proofs that for high frequency mode of circumferential wave number m nonlinear interaction exits only with gravity wave modes of circumferential wave number zero or $2m$. Multi-scale analysis reveals that appearance of such phenomenon is due to Hopf bifurcation of the dynamic system. Curves of critic excitation force with respect to excitation frequency are obtained by analysis. Theoretical results show good qualitative and quantitative agreement with experimental observations.

1. Introduction

Large amplitude low frequency gravity water waves generated by a high frequency excitation at a water-filled beaker is firstly discovered by Huntley [1]. Wang et al. observed the same phenomenon when doing an experimental research on Dragon Washbasin [2]. Systematic experiments are then carried out by Wang et al. to investigate the frequency and critic magnitude of excitation required for appearance of this phenomenon, the relationships between aroused gravity waves and high frequency natural modes and also transition rules of different gravity waves, which are discussed extensively in part I of this work [3].

Mahony and Smith [4] propose an idealized model to explain this called "spatial resonant" phenomenon. They studied a simple 2-dimensional (2-D) rectangular enclosure half-filled with water and then driven as an acoustic resonance tube. Considering nonlinear aerial and water wave interactions, they developed an equation governing the critic acoustic pressure amplitude with respect to excitation frequency when this phenomenon occurs, which was validated by experimental results of Huntley [1] and Franklin et al. [5]. Huntley [6] extends Mahony and Smith's model to real three-dimensional (3-D) cases. Miles [7] examined the conjecture of Mahony and Smith's model by analyzing Hamilton equations of the air fluid coupled system. Miles however declared that the nonlinear quadratic coupling between aerial and surface waves is $O(\varepsilon)$, rather than $O(1)$, and the putative resonance is either impossible or realized only at higher order. Hsieh [8] presented a parametric resonance model and demonstrated numerically the existence of low frequency gravity waves.

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Noticing that previous researches concentrated only on the water waves in the fluid domain excited by prescribed vessel motions cannot give precise description of this nonlinear water-shell interaction problem, Zhou et al. [9] establish a model of nonlinear modal interactions based on variational principle of fluid–structure coupled system to analysis this phenomenon. A set of nonlinear equations involving four modes interactions is then obtained. Unlikeness Mile's [7] assertion, the resulted mode coupling order in [9] is O(1) in mode interaction equations. This can be explained that because the ratio of natural frequencies of two modes is $O(\varepsilon^{-1})$, differential of the Lagrangian can produce higher coupling order. Results from numerical simulation of the equations in [9] reveal good agreement with experimental observations. The objective of this paper is to further explore analytically the mechanism of the experimental phenomenon through a concise theoretical model. Based on the theory given in [9], a two mode interaction model is therefore adopted to assist analysis for the major factors affecting nonlinear interactions and the stabilities of the dynamic system. Compared with four-mode approximation used in [9], two-mode approximation is more convenient to study mathematically the rules of mode interaction and to make clear the key parameters causing the nonlinear resonance.

The method of multiple scales is adopted to analysis the nonlinear equations since straight forward analytical results can be obtained by closed-form approximations for primary and secondary resonances. By application the method of multiple scales, parameters for onset of nonlinear bifurcations are to be explored which correspond to curve of critic excitation amplitude with respect to excitation frequency measured in experiments.

2. Two modes coupling equations

The fluid-shell coupled system to be considered comprises an elastic cylindrical thin shell and partially filled water as shown in Fig. 1. The lower bottom of the shell is fixed and the upper bottom is free. A horizontal sinusoidal excitation with amplitude F and angular frequency ω is applied at a point on the shell. Denote shell radius, height, and the quiescent water depth as a, b, and d, respectively. Let r, θ and z be a Cylindrical coordinate system fixed with the shell so that r and θ lie in the undisturbed free surface and z is upward and positive away from the bottom of the shell.

Denote the displacements of the shell middle surface in directions of z, θ, r as u, v, w, respectively. It is reasonable to assume the governing equation of shell as linear, since the displacement of shell measured by experiments is sufficiently small compared with shell thickness. The displacement of the water surface is comparatively large, thus the nonlinearity would be considered due to the nonlinear boundary conditions for the fluid.

Let S_F and S_{F^0} be kinematical and quiescent free surface of the fluid. With η the height between kinematical free surface and quiescent free surface, the free surface S_F can be described by

$$S_F(z, r, \theta, t) = z - \eta(r, \theta, t) = 0 \tag{1}$$

Let S_W and S_{W^0} be kinematical and quiescent wet surface (contact surface of fluid and shell). With very small w, the wet surface S_W can be described by

$$S_W(r, \theta, z, t) = r - a - w(z, \theta, t) = 0 \tag{2}$$

The water is treated as inviscid, incompressible and its motion to be irrotational so that its velocity can be derived from a potential function $\phi(r, \theta, z)$. Variational principle for fluid dynamics in Eulerian description was developed by Luck [10] and Seliger et al. [11]. Variational principle (see, Eq. (4.26) in [12]) of nonlinear dynamical fluid–solid interaction systems was developed by Xing and Price [12]. The application of this variational principle gives a function describing the shell-water interaction system shown in Fig. 1 as follows

$$\delta J(\phi, \eta, u, v, w) = \delta \int_{t_1}^{t_2} L dt \equiv \delta \int_{t_1}^{t_2} (L_f + L_s) dt = \delta W \tag{3}$$

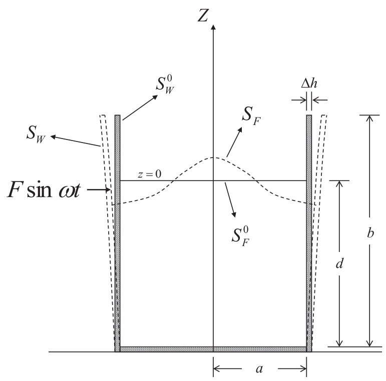

Fig. 1. Schema of elastic shell and water interaction system.