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Communications in Nonlinear Science and Numerical Simulation 10 (2005) 329–340

Communications in Nonlinear Science and Numerical Simulation

www.elsevier.com/locate/cnsns

Global behaviors of a generalized periodic impulsive Logistic system with nonlinear density dependence

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Received 24 September 2002; received in revised form 23 February 2003; accepted 20 March 2003
Available online 23 October 2003

Abstract

The global behaviors of a generalized periodic impulsive Logistic system with nonlinear density dependence are studied. Conditions for the existence and global attractivity of positive periodic solution are obtained via the method of comparison and Liapunov function. The corresponding results for the periodic impulsive Logistic system, which are dependent on solving the system, are extended.

PACS: 02.30.Hq; 02.60.Lj; 02.10.Jf
Keywords: Impulse; Logistic system; Global attractor; Periodic solution

1. Introduction

The impulsive differential equations may express naturally real-world simulation processes with state changing of jumps. These processes occur in a wide variety of applications such as the theoretical physics, population dynamics, biotechnology, economics, drug administration, etc.

Recently, there has been a significant development in theory of impulsive differential equation, especially in the area where impulses are at fixed time. However, investigations are mainly focused on the basic theories. For example [1,2] established conditions for the existence and absence of the pulse phenomenon, [3,4] studied the existence and continuability of solutions, and [5–10] devoted to the oscillating properties of solutions.

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1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2003.03.001

Further results of the impulsive differential equations arisen from applications are not easily obtained due to numerous theoretical and technical difficulties except that in some cases the models can be rewritten as simple discrete-time mapping or difference equations when the corresponding continuous models can be solved explicitly, e.g., [11,12]. Even for the simple impulsive periodic Logistic equations, only conditions for the existence and local stability of a unique positive periodic solution were given in [13]. And the method was still based on solving the equation.

In this paper, we will study the following generalized periodic Logistic system with periodic impulses and nonlinear density dependence.

$$\begin{cases} \dot{y}(t) = y(t)(r(t) - \sum_{i=1}^{n} a_i(t)y^{\alpha_i}(t)), & t \neq t_k, \ k \in \mathbb{N}, \\ \Delta y(t_k) = b_k y(t_k), & k \in \mathbb{N}, \end{cases} \tag{1}$$

where $t_0 \triangleq 0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, $\Delta y(t_k) = y(t_k^+) - y(t_k)$, $r(\cdot)$, $a_i(\cdot) \in PC[\mathbb{R}, \mathbb{R}]$ and $PC[\mathbb{R}, \mathbb{R}] = \{\phi : \mathbb{R} \mapsto \mathbb{R}, \phi$ is continuous for $t \neq t_k$, $\phi(t_k^+) = \lim_{t \to t_k^+} \phi(t)$ and $\phi(t_k^-) = \lim_{t \to t_k^-} \phi(t)$ exist and $\phi(t_k) = \phi(t_k^-)$, $k \in \mathbb{N}\}$. α_i are positive constants, $i = 1, \ldots, n$. The intrinsic rate $r(t)$ and the density dependence coefficients $a_i(t)$, $i = 1, \ldots, n$, are supposed to be time-varying, since the species and the environment change with time. And particularly, the variation is usually periodic. For biological senses, we suppose the following conditions hold through out this paper.

(H1) (1) is ω-periodic, i.e.,

$$r(t + \omega) = r(t), a_i(t + \omega) = a_i(t), \quad t \in \mathbb{R}, \ i = 1, \ldots, n$$

and there is a $q \in \mathbb{N}$ such that there are q t_ks in the interval $(0, \omega)$ and

$$t_{k+q} = t_k + \omega, \quad b_{k+q} = b_k, \ k \in \mathbb{N}.$$

(H2) $r(t) \geqslant 0$, $\inf_{t \in [0,\omega]} a_i(t) > 0$.

(H3) $1 + b_k > 0$, $b_k \neq 0$, $k \in \mathbb{N}$.

System (1) describes the variation of the population number $y(t)$ of an isolated species in a periodically varying environment. The intrinsic rate of change $r(t)$ is related to the periodically varying possibility of regeneration of the species, the density dependence is nonlinear and coefficients $a_i(t)$, $i = 1, \ldots, n$, are related to the periodic changes of resources maintaining the evolution of the population. When $b_k > 0$, the impulse stands for planting of the species, while $b_k < 0$ stands for harvesting. By the basic theories of impulsive differential equations in [13,14], (1) has a unique solution $y(t) = y(t, y_0) \in PC[\mathbb{R}, \mathbb{R}]$ for each initial value $y(0) = y_0 \in \mathbb{R}_+$ and further $y(t) > 0, t \in \mathbb{R}_+$ if $y(0) = y_0 > 0$.

The purpose of this paper is to study the global behaviors of system (1). We will also consider the following related system:

$$\dot{x}(t) = x(t)\left(r(t) - \sum_{i=1}^{n} q_i(t)x^{\alpha_i}(t)\right), \quad t \neq t_k, \ k \in \mathbb{N}, \tag{2}$$

where

$$q_i(t) = a_i(t) \prod_{0 < t_k < t} (1 + b_k)^{\alpha_i}, \quad i = 1, \ldots, n.$$