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On the global exponential stability of impulsive functional differential equations with infinite delays or finite delays ☆

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ABSTRACT

This paper considers the impulsive functional differential equations with infinite delays or finite delays. Some new sufficient conditions are obtained to guarantee the global exponential stability by employing the improved Razumikhin technique and Lyapunov functions. The result extends and improves some recent works. Moreover, the obtained Razumikhin condition is very simple and effective to implement in real problems and it is helpful to investigate the stability of delayed neural networks and synchronization problems of chaotic systems under impulsive perturbation. Finally, a numerical example and its simulation is given to show the effectiveness of the obtained result in this paper.

1. Introduction

In the past several years, the theory of impulsive functional differential equations has attracted the interest of many researchers due to their wide applications in science and technology and various interesting results have been reported. For details, see [1–18] and references therein. However, some undesirable difficulties with the theory and application to real problems are encountered simultaneously. For instance, the authors [7,9,12] obtained some sufficient conditions to guarantee the uniform stability and uniform asymptotic stability of impulsive functional differential equations with infinite delays. However, to construct such suitable Lyapunov functional to satisfy those conditions in [7,9,12] is very difficult. Moreover, those results cannot guarantee the exponential stability. In practice such as designs and applications of neural networks to solve various optimization problems, neural networks have to be designed such that there is only one equilibrium point and this equilibrium point is exponentially stable with a fast exponential convergent rate, so as to avoid the risk of having spurious equilibria and being trapped at local minima [19,20]. Also, the exponential stability property is particularly important when the exponential convergence rate is used to determine the speed of neural computations [21,22]. Thus, it is necessary and important to investigate exponential stability and to estimate the exponential convergence rate of impulsive functional differential equations from the view of both theory and application. In [12], the authors obtained some Razumikhin-type results on exponential stability for impulsive functional differential equations with infinite delays. However, these results are only valid for some specific systems due to the restrictive requirements such as complicated Razumikhin

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condition and rigorous impulsive interval. Therefore, techniques and methods for impulsive functional differential equations with infinite delays should be developed and explored.

Motivated by the above discussions, in this paper, we further. investigate the exponential stability of impulsive functional differential equations with infinite delays. By employing the improved Razumikhin technique and Lyapunov functions, some new conditions ensuring the global exponential stability are obtained. The result can complement and improve some recent works [7–13]. Moreover, the required Lyapunov functions in this paper are much easier to construct. Finally, a numerical example and its simulation is given to show the effectiveness of the obtained theoretical result.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{Z}_+$ the set of positive integers and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $|\bullet|$. $\mathbb{K} = \{a \in C(\mathbb{R}_+, \mathbb{R}_+)|\ a(0) = 0$ and $a(s) > 0$ for $s > 0$ and a is strictly increasing in $s\}$. For any $t \geqslant t_0 \geqslant 0 > \alpha \geqslant -\infty$, let $f(t, x(s))$ where $s \in [t+\alpha, t]$ or $f(t, x(\cdot))$ be a Volterra type functional. In the case when $\alpha = -\infty$, the interval $[t+\alpha, t]$ is understood to be replaced by $(-\infty, t]$. Let $\mathbb{C}$ be an open set in $PC([\alpha, 0], \mathbb{R}^n)$, where $PC([\alpha, 0], \mathbb{R}^n) = \{\varphi : [\alpha, 0] \to \mathbb{R}^n$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+), \varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$. Define $PCB(t) = \{x_t \in \mathbb{C} : x_t$ is bounded$\}$. For $\varphi \in PCB(t)$, the norm of φ is defined by $||\varphi|| = \sup_{\alpha \leqslant \theta \leq 0} |\varphi(\theta)|$.

Consider the following impulsive functional differential equations with infinite delays:

$$\begin{cases} x'(t) = f(t, x(\cdot)), & t \geqslant t_0,\ t \neq t_k, \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(t_k, x(t_k^-)), & k \in \mathbb{Z}_+, \end{cases} \tag{1}$$

where the impulse times t_k satisfy $0 \leqslant t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k \to +\infty} t_k = +\infty$ and x' denotes the right-hand derivative of x. $f \in C([t_{k-1}, t_k) \times \mathbb{C},\ \mathbb{R}^n)$, $f(t, 0) = 0$ and $I_k(t, x) \in C([t_0, \infty) \times \mathbb{R}^n, \mathbb{R}^n)$. For each $t \geqslant t_0$, $x_t \in \mathbb{C}$ is defined by $x_t(s) = x(t+s),\ s \in [\alpha, 0]$. $I_k(t_k, 0) = 0,\ k \in \mathbb{Z}_+$.

For any $\sigma \geqslant t_0$ and $\phi \in \mathbb{C}$, the initial condition for system (1) is given by

$$x_\sigma = \phi, \quad \alpha \leqslant s \leqslant 0. \tag{2}$$

Remark 2.1. In [14,15], the authors have obtained some results on existence and uniqueness of impulsive functional differential equations with infinite delays. In this paper, we always assume that the solution for the initial problem (1) and (2) does exist and is unique which will be written in the form $x(t, \sigma, \phi)$. Since $f(t, 0) = 0,\ I_k(t_k, 0) = 0, k \in \mathbb{Z}_+$, then $x(t) \equiv 0$ is a solution of (1) and (2), which is called the trivial solution. Moreover, we will only consider the solution $x(t, \sigma, \phi)$ of (1) and (2) which can be continued to ∞ from the right of σ.

We also need the following definitions:

Definition 2.1. The function $V : [\alpha, \infty) \times \mathbb{C} \to \mathbb{R}_+$ belongs to class ν_0 if

(H_1) V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{C}$ and $\lim_{(t, \varphi_1) \to (t_k^-, \varphi_2)} V(t, \varphi_1) = V(t_k^-, \varphi_2)$ exists;

(H_2) $V(t, x)$ is locally Lipschitzian in x and $V(t, 0) \equiv 0$.

Definition 2.2. Let $V \in \nu_0$, for any $(t, \psi) \in [t_{k-1}, t_k) \times \mathbb{C}$, the upper right-hand Dini derivative of $V(t, x)$ along the solution of (1) and (2) is defined by

$$D^+V(t, \psi(0)) = \limsup_{h \to 0^+} \frac{1}{h}\{V(t+h, \psi(0) + hf(t, \psi)) - V(t, \psi(0))\}.$$

Definition 2.3. The trivial solution of (1) is said to be globally weakly exponentially stable if there exist functions $\alpha_1, \alpha_2 \in \mathbb{K}$ and constants $\lambda > 0, \mathscr{M} \geqslant 1$ such that for any initial value $x_\sigma = \phi \in PCB(\sigma),\ \alpha_1(|x(t)|) < \mathscr{M}\alpha_2(||\phi||)\ e^{-\lambda(t-\sigma)},\ t \geqslant \sigma$. Especially when $\alpha_1(s) = \alpha_2(s) = s$, it is usually called to be globally exponentially stable.

3. Main results

Now we shall establish a new exponential stability theorem for impulsive functional differential Eqs. (1) and (2) by employing the improved Razumikhin technique and Lyapunov functions.

Theorem 3.1. *Assume that there exist functions* $w_1,\ w_2 \in \mathbb{K},\ c \in C(\mathbb{R}_+, \mathbb{R}_+),\ p \in PC(\mathbb{R}_+, \mathbb{R}_+),\ V(t, x) \in \nu_0$, *and some constants* $q > 1,\ \gamma > 0,\ \beta_k \geqslant 0,\ k \in \mathbb{Z}_+$ *such that*