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An alternating iterative algorithm for image deblurring and denoising problems ☆

Si Wang, Ting-Zhu Huang *, Jun Liu, Xiao-Guang Lv

School of Mathematical Sciences/Institute of Computational Science, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China

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A B S T R A C T

In this paper, a modified l_1 minimization model for image delurring and denoising problems is considered. To solve the proposed l_1 minimization model, we present an efficient alternative iterative algorithm in which the fast iterative shrinkge-thresholding method (FISTA) and the well known dual approach for solving the denosing problems are alternately employed. Besides, we prove the convergence of the proposed algorithm. Numerical results demonstrate the efficiency and viability of the proposed algorithm to restore the degraded images.

1. Introduction

Image restoration is one of the most fundamental task in image processing and plays an important role in various areas of applied sciences [1]. In this paper, we consider a common discrete imaging degradation model which can be expressed as follows:

$$g = Hf + n. \tag{1}$$

Here, $f \in R^{n^2}$ is an original image, $H \in R^{n^2 \times n^2}$ is a blurring matrix which we assume to be known, $n \in R^{n^2}$ is an additive Gaussian white noise, and g is the observed image. Without loss of generality, we assume that the underlying images have square domains. Our aim is to recover f from g, which is known as deconvolution or deblurring. A general approach to compute a useful approximation solution of (1) is replacing system (1) by a better-conditioned nearby system. This replacement is known as the regularization. One of the commendable regularization method is Tikhonov regularization, in which the linear system (1) is replaced by a minimization problem, e.g.,

$$\min_f \{\|Hf - g\|_2^2 + \lambda \|Lf\|_2^2\}, \tag{2}$$

where λ is a positive regularization parameter which provides the trade-off between fidelity to the measurements and noise sensitivity, the matrix L is a regularization operator. We often choose L as the identity matrix or a matrix approximating the first

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* Corresponding author. Tel.: +86 28 61831608; fax: +86 28 61831280.

E-mail addresses: uestcsiwang@163.com (S. Wang), tingzhuhuang@126.com, tzhuang@uestc.edu.cn (T.-Z. Huang), sino.roy@163.com (J. Liu), little006@126.com (X.-G. Lv).

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or second order derivation operator [2,3]. Throughout this paper, $\|\cdot\|_2$ denotes the Euclidean norm. It is well known that Thikhonov-like regularization tends to make images overly smoothed and often fails to adequately preserve important image attributes such as sharp edges. More recently, l_1 regularization has attracted a considerable attention which seeks to find the solution of

$$\min_f \{\|Hf - g\|_2^2 + \lambda\|Lf\|_1\}, \tag{3}$$

where $\|Lf\|_1$ stands for the sum of the absolute values of the components of Lf. More relative works on promoting the use of l_1 regularization, as well as its relevance to other research areas, can be found in the recent work [4]. Many important problems in imaging science can be viewed as l_1 regularization optimization problems. The distinctive advantage of the l_1-based regularization is that l_1 regularization is less sensitive to outliers. The presence of the l_1 term encourages small components of f to become exactly zero, thus inducing the sparsity in the optimal solution of (3). One of the most successful l_1 regularization model proposed by Rudin et al. [5] is the total variation (TV) based regularization scheme model (the ROF model). A discrete version of the unconstrained TV deblurring problem [6] is given by

$$\min_f \{\|Hf - g\|_2^2 + \alpha\|f\|_{TV}\}, \tag{4}$$

where α is a positive regularization parameter and $\|\cdot\|_{TV}$ is the discrete TV regularization term. The discrete gradient operator $\nabla : R^{n^2} \to R^{n^2}$ is defined by

$$(\nabla f)_{j,k} = ((\nabla f)^x_{j,k}, (\nabla f)^y_{j,k})$$

with

$$(\nabla f)^x_{j,k} = \begin{cases} f_{j+1,k} - f_{j,k} & \text{if } j < n, \\ 0 & \text{if } j = n, \end{cases}$$

$$(\nabla f)^y_{j,k} = \begin{cases} f_{j,k+1} - f_{j,k} & \text{if } k < n, \\ 0 & \text{if } k = n, \end{cases}$$

for $j,k = 1,\ldots,n$. Here $f_{j,k}$ refers to the $((k-1)n+j)$th entry of the vector f (it is the (j,k)th pixel location of the image). The discrete total variation of f is defined by

$$\|f\|_{TV} = \sum_{1\leqslant j,k\leqslant n} \sqrt{|(\nabla f)^x_{j,k}|^2 + |(\nabla f)^y_{j,k}|^2}. \tag{5}$$

The definition in Eq. (5) is often referred to as isotropic total variation [7]. An alternative, approximative definition of the TV functional also exists, and is handled by some algorithms primarily to simplify the problem. This alternative definition is known as anisotropic TV

$$\|f\|_{TV} = \sum_{1\leqslant j,k\leqslant n} |(\nabla f)^x_{j,k}| + |(\nabla f)^y_{j,k}|.$$

Throughout this paper, we only focus on the use of the isotropic formulation of TV. Actually, it is also suitable for anisotropic TV. The ROF functional, despite its simple form, has been proved to be very difficult to minimize by conventional methods due to the TV functional is not differentiable. A number of numerical methods have been proposed for solving (4). These methods include fixed point techniques, smoothing approximation methods, splitting methods, dual methods, primal–dual Newton-based methods, gradient-based algorithms, second-order cone programming, graph-based approaches and interior point algorithms, see for instance, [5,8–14], and references therein. In order to use $\|f\|_{TV}$ exactly in formulation and avoid a numerical difficulty, in particular, Chambolle [15] proposed a globally convergent gradient-based algorithm when H is the identity matrix and which was shown to be faster than primal-based schemes.

In image restoration applications, l_1 regularization is less sensitive to sharp edges and promote sparse solutions. Motivated by this, in this paper, we study an effective l_1 minimization model for image restoration. The proposed unconstrained l_1 regularized minimization problem is given by

$$\min_{f,u} \mathcal{J}(f,u) \equiv \min_{f,u} \|Hf - g\|_2^2 + \alpha_1\|f - u\|_2^2 + \alpha_2\|f\|_1 + \alpha_3\|u\|_{TV}, \tag{6}$$

where α_1, α_2 and α_3 are positive regularization parameters. The main difference between (4) and (6) is that fitting terms $\|f-u\|_2^2$ and $\|f\|_1$ are added in the new l_1 regularized minimization problem. It is obvious that

$$\min_{f,u} \|Hf - g\|_2^2 + \alpha_1\|f - u\|_2^2 + \alpha_2\|f\|_1 + \alpha_3\|u\|_{TV} = \min_u \min_f \{\|Hf - g\|_2^2 + \alpha_1\|f - u\|_2^2 + \alpha_2\|f\|_1\} + \alpha_3\|u\|_{TV}. \tag{7}$$

In (6) or (7), we can interpret the TV minimization scheme to denoise the restored image f. The main advantage of the proposed method is that l_1 and TV norm are used in the image restoration process. Therefore, the proposed method is able to preserve the edge quite well. We use an alternating minimization algorithm to solve the proposed l_1 minimization problem. Our numerical examples show the effectiveness of the proposed approach. This method is competitive with those restored by the existing total variational methods.