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Modulation of synchronization dynamics in a network of self-sustained systems

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This paper addresses the combined modulatory effects of non-nearest neighbor oscillators and local injection on synchronized states dynamics with their corresponding stability boundaries in a network of self-sustained systems. The Whittaker method and Floquet theory are used to predict analytically the stability of these states for identical and non-identical coupling parameters. Charts revealing the modulation of synchronized states and their stability boundaries at the second order of interaction in the cases of identical and non-identical coupling parameters are constructed with and without an external signal locally injected in the network. Numerical simulations validate and complement the results of analytical surveys. The limits of the stability regions are numerically explored when a small amount of Gaussian white noise is also injected in the network.

1. Introduction

Coupled nonlinear dynamical systems have become a topic of growing interest since they show very rich phenomena such as synchronization. The synchronization of nonlinear oscillations occurring in networks, such as arrays or rings, is a rather attractive topic due to the enormous variety of potential applications. For instance, arrays of coupled nonlinear oscillators have been used for the description of Josephson-junctions [1], multimodes lasers [2,3], relativistic magnetrons [4]. Furthermore, arrays of oscillators also arise in studies of biological rhythms of the heart [5], nervous system [5,6], intestines [7], pancreas [8] and other biological systems [9–12]. The case of ring coupling topology is also of interest. For example, rings of two neuronal systems have been used to study the mixed inhibitory and excitatory circuit coordinating the motion of locust wings during flight [13]. It has also been demonstrated that in quadrupedal mammals, the four oscillators controlling the limb may be coupled in some form of ring [14,15]. A previous study has investigated different states of synchronization in a ring of mutually coupled self-sustained electrical oscillators [16]. Properties of the variational equations of stability have been utilized to investigate the dynamics of the ring and a stability map displaying domains of synchronization to a locally injected external excitation has been reported. Recently, these results have been validated experimentally and the consequences of parameter mismatch have been also emphasized [17]. But, these studies [16,17] and others [18–20] have considered only the influence of nearest neighbors coupling on stability boundaries using both analytical and numerical investigations. Thus, we want to investigate in this paper the modulatory effect of non-nearest neighbors and the local injection on synchronized states and their corresponding stability boundaries. We analyze first the stability of the synchronization process when both first and second order couplings are considered. Later, we address the question of the

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synchronization conditions when an external signal is locally injected in the network. The paper is organized as follows. In Section 2, we introduce the system configuration and problem statement. In Section 3, we investigate the impact of the range of interaction on the stability boundaries for identical and nonidentical coupling parameters. Section 4 deals with the influence of a local injection for identical and nonidentical coupling parameters. Conclusions are given in Section 5.

2. System configuration and problem statement

The model shown in Fig. 1 is a ring of N identical mutually coupled self-sustained electrical systems where each unit is modeled as van der Pol oscillator (see Fig. 2). Each van der Pol oscillator consists of a nonlinear resistor NLR, an inductor L and a condenser C, all connected in parallel. The coupling between the units is realized through an inductor L_c. Experimentally, the network can be built using $TL-082$ operational amplifiers and $AD-633$ multipliers as reported previously [17].

The volt-ampere characteristic of the nonlinear resistor for the kth unit is expressed by a symmetric cubic nonlinearity, which is illustrated by

$$i_k = -e_1 V_k + e_3 V_k^3, \quad e_1, e_3 > 0;\ 1 \leqslant k \leqslant N. \tag{1}$$

This form of nonlinearity was introduced by van der Pol who had considered a lumped oscillator with two degrees of freedom to discuss simultaneous multi-mode oscillations [21,22]. In such situation, the oscillator traces a particular path through phase space, and if some perturbation excites it out of its accustomed rhythm, it soon returns to its former path. Oscillators that have a standard waveform and amplitude to which they return after small perturbations are known as limit-cycle oscillators (e.g. van der Pol oscillators). As shown in the appendix, the network is described by the following set of second order non-dimensional nonlinear differential equations:

$$\ddot{x}_k - \mu(1 - x_k^2)\dot{x}_k + x_k = K_1(x_{k+1} - 2x_k + x_{k-1}) + K_2(x_{k+2} - 2x_k + x_{k-2}), \tag{2}$$

where x_k stands as the voltage amplitude of the kth oscillator and μ is a positive coefficient of nonlinearity. K_1 and K_2 are the coupling strengths of the first and second nearest neighbors respectively. The model assumes the boundary conditions cyclic

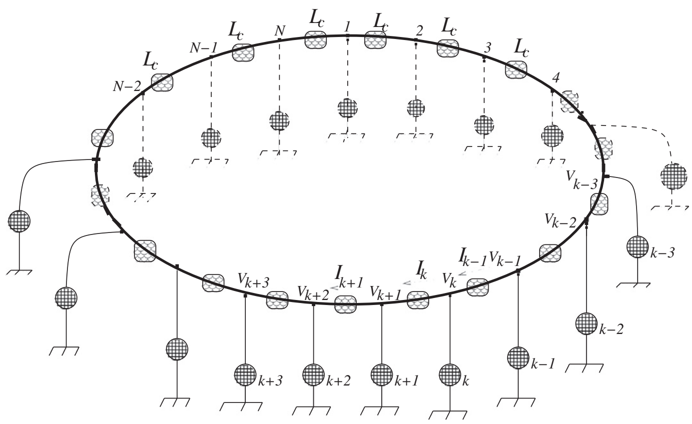

Fig. 1. Network of N mutually coupled self-sustained electrical oscillators.

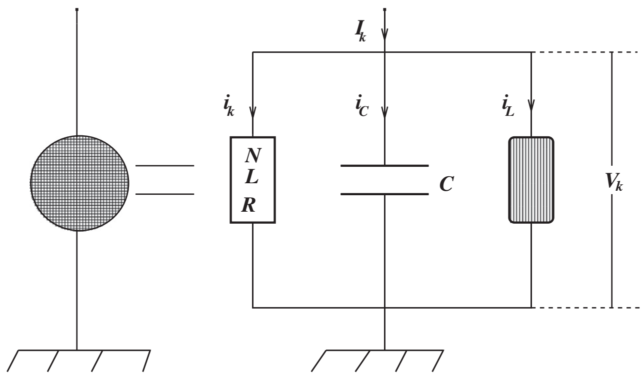

Fig. 2. Model of a self-sustained electrical oscillator.