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A probabilistic theory of random maps

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Abstract

We present a probabilistic theory of random maps with discrete time and continuous state. The forward and backward Kolmogorov equations as well as the FPK equation governing the evolution of the probability density function of the system are derived. The moment equations of arbitrary order are derived, and the reliability and first passage time problem are also studied. Examples are presented to demonstrate the application of the theoretical development. Numerical solutions including the time histories of moment evolution, steady state probability density function, reliability and first passage time probability density function for time discrete random maps are included. The present work compliments the existing theory of continuous time stochastic processes.
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1. Introduction

A series of seminal papers by Ma and Caughey presented rigorous stability analyses of linear and nonlinear discrete random maps [1–3]. The stability conditions for higher order moments have been derived in [2]. In [3] the relationship between mean stability and various types of stability definitions has been explored. The references cited above have primarily focused on the stability of random maps. Little has been done in the literature regarding the probability density

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evolution and studies of reliability and first passage time of random maps. These subjects are the focus of the present work.

There are many studies on discrete stochastic systems where the states are discrete while the time is continuous, or the probability of random events is discrete. Discrete state stochastic systems find applications to the study of software failure [4] and quantum mechanics [5–7]. Tagliani studied the existence and stability of the solutions of moment specification problems for discrete probability densities using a modified version of maximum entropy principle [8,9]. Hanzon and Ober [10] studied a discrete event system that have discrete states with geometric or Poisson distributions. Garg et al. [11] established a framework for a discrete stochastic systems where language operators are defined. Feng investigated a time discrete model of stochastic friction system using Poincaré maps [12].

In this study, we deal with random maps with discrete time and continuous state, the same type of systems considered in [1–3]. For continuous time stochastic systems, the forward and backward Kolmogorov equations as well as the Fokker–Planck–Kolmogorov (FPK) equation govern the evolution of probability density of the process [13,14]. Here, we derive the counterpart of these equations for discrete time random maps. The formulations of reliability and first passage time problem are then presented by using these equations. The present study, therefore, complements the theory for continuous time stochastic systems.

The paper is organized as follows. In Section 2, we derive the forward Kolmogorov and FPK equations for time discrete random maps. The moment equations for random maps are obtained using the forward Kolmogorov equation. The backward Kolmogorov equation is derived in Section 3. The reliability, first passage time problem, and Pontryagin–Vitt type moment equations of the first passage time are then investigated. The theory is demonstrated with examples. A 1-D nonlinear map is examined in Section 4. In Section 5, we study the moment dynamics of a 2-D random map. Using the backward Kolmogorov equation, we investigate the reliability and first passage time probability of the 2-D system subject to external random excitations.

2. Mathematical theory

Consider a discrete time random map as follows

$$\mathbf{Y}(n+1) = \Phi(\mathbf{Y}(n), n, \mathbf{W}) = \mathbf{S}(\mathbf{Y}(n), n) + \Gamma, \tag{1}$$

where n is the discrete time index, $\mathbf{Y} \in \mathbf{R}^p$ is the state vector, $\Phi(\mathbf{Y}(n), n, \mathbf{W})$ is a generic random map and $\mathbf{W} \in \mathbf{R}^q$ is a vector of random parameters. A special form of the map is also included where additive random contributions $\Gamma \in \mathbf{R}^p$ and a parametric random map $\mathbf{S} : \mathbf{R}^p \times R^1 \mapsto \mathbf{R}^p$ are explicitly shown. Assume that the initial condition of the mapping system is specified by a known function $p(\mathbf{y}_0, n_0)$. The PDF of $\mathbf{Y}$ at time n conditional on the initial condition $(\mathbf{y}_0, n_0)$ is denoted by $p(\mathbf{y}, n|\mathbf{y}_0, n_0)$.

Note that Eq. (1) can also be viewed as a closed loop control system. Therefore, the present study can have a broad range of applications to stochastic digital controls. Next, we shall derive equations governing the evolution of $p(\mathbf{y}, n|\mathbf{y}_0, n_0)$.

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