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Solitary wave in a nonlinear elastic structural element of large deflection ☆

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Abstract

General form nonlinear governing equations for the wave traveling in a nonlinear elastic structural element of large deflection are derived in the present research. An asymptotic solution of solitary wave in the elastic element is derived and investigated by means of a modified complete approximate method. Numerical computations for the solution are carried out. Characteristics of the solitary wave are investigated with various system parameters and initial conditions. Shapes and the propagation of the nonlinear elastic wave are also illustrated with figures. Based on the theoretical and numerical analyses of the research, quantitative conclusions are obtained for the wave motion of the elastic structural element.

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1. Introduction

Recently there have been considerable interests in the occurrence of chaotic states in deterministic systems. In solid mechanics, many researchers have studied the bifurcation and chaotic

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motion of the nonlinear elastic elements or systems with/without large deflection [1–7]. There are two main concerns in the systems of nonlinear differential equations for the waves in a nonlinear elastic element: namely, solitons and chaos of the solutions of the governing equations. One of the interesting progresses found in the current literature is a study with perturbation approach on the soliton propagating along a circular rod of a hyperelastic material with variable cross-sections and density [8]. It is found in the research that a soliton wave of the rod may develop into a solitary wave. A significant contribution was made by Luo et al. [9] on predicting the chaotic response of a nonlinear rod. Chaotic and bifurcation conditions were provided. Investigations on the nonlinear behavior of a pinned Sine–Gordon soliton and the nonlinear Schrödinger systems are also reported [10]. Though more archival publications dealt with the propagation of the nonlinear wave in the structural elements taking into considerations of physical nonlinearity and material nonlinearity are found in the literature, systematic and thorough analyses on the behavior of the wave in highly nonlinear structural elements are still in need.

The present research intends to investigate the wave motion in a nonlinear elastic structural element with large deflection subjected to an external exertion. The KdV–mKdV equation is to be established for the wave motion. An asymptotic solution of solitary wave is to be derived with utilization of a modified complete approximate method. With the governing equation of the wave motion in the element and the solution derived, the characteristics of the nonlinear elastic wave of the element can be analyzed theoretically and numerically. Properties of the wave propagation and the effects of the system parameters of the elastic element and the influences of the initial conditions to the characteristics of the wave motion are investigated in details. It is anticipated that the results of the present research can be furthered to the analytical and numerical investigations for the wave propagations of nonlinear elastic solid elements.

2. Development of governing equation

Consider a uniform nonlinear elastic circular structural element of infinite length as shown in Fig. 1, where ρ is the mass per unit length and R is the radius of the element.

The structural element so defined is axial-symmetric, cylindrical coordinates are therefore convenient for implementation. Assume that the plane section hypothesis holds true in the case that the element is exerted by an axial compression or an extension impulse. Additionally, for the sake of clarification in developing the equations of motion of the element, the following fundamental hypotheses stand true.

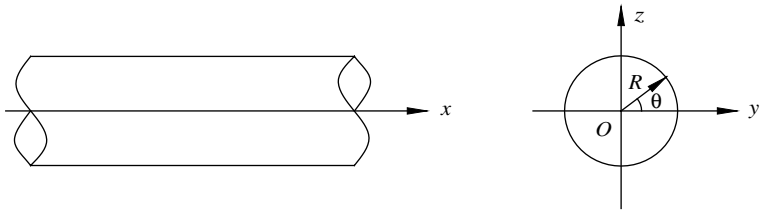

Fig. 1. The structural element with infinite length.