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Numerical bifurcation analysis of the travelling waves on a falling liquid film

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Abstract

Experiments for films flowing down a vertical substrate demonstrate a wide spectrum of wave regimes which are strongly dependent on flow conditions and liquid properties. This paper presents a comprehensive numerical bifurcation analysis of the steady travelling waves on a falling film described by an equation containing two parameters. It is shown that the sensitivity of observed wave regimes to physical parameters is related to a set of bifurcations associated with the variation of these parameters.

1. Introduction

Film flow down a vertical plane at moderate flow rates, or a falling film, has been considered in numerous experimental and theoretical investigations. Falling films demonstrate a wide variety of flow regimes, which are very sensitive to flow conditions. The first systematic experimental investigations [1] demonstrated the existence of two principal wave types: periodic sinusoidal waves and solitary waves, travelling with constant velocity. These so-called *regular waves* can take on different shapes, amplitudes and velocities depending on flow conditions.

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The principal methods of theoretical investigation based on use of a thin layer approximation were suggested in [2]. Numerical investigations have determined numerous types of waves in the framework of this approximation [3,4] as well as in an equation derived in [5] with the help of the assumption of a parabolic velocity profile, see, for instance, [5–11]. Several reviews of experimental and theoretical investigations have also been given in [12–15].

The investigation of singularities of the system considered in this paper is important because generalizations have been developed for film flows with additional effects: Marangoni effects for heated films [16] and films containing surfactant [17,18], film flow over a spinning disk [19], a falling power-law fluid film [20] and others. All these flows are essentially distinct at finite values of the principal film parameter δ. Hence, singularities related to variation of this parameter, together with effects of other similarity parameters, determine a wide variety of film flows.

It is worthy of note that all of the principal results relating to the falling film problem have been achieved with the help of intensive application of numerical methods. The reason is due to the strongly non-linear properties of the travelling waves: construction of a two-parameter manifold of periodic solutions requires up to 256 harmonics for a single point.

2. Mathematical model

2.1. The evolution system

The evolution system derived in [5] has the form

$$\begin{aligned}&\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=0,\\&\frac{\partial q}{\partial t}+\frac{6}{5}\frac{\partial}{\partial x}\left(\frac{q^2}{h}\right)=\frac{1}{5\delta}\left(h\frac{\partial^3 h}{\partial x^3}+h-\frac{q}{h^2}\right),\end{aligned} \tag{1}$$

where x and t are space and time coordinates, h is film thickness and q is local flow rate. The model (1) contains a single similarity parameter

$$\delta=\frac{1}{45\nu^2}\left(\frac{\rho H_c^{11}g^4}{\sigma}\right)^{1/3}, \tag{2}$$

where ρ, ν and σ are the density, the viscosity and the surface tension of the liquid, g is the gravity; H_c is a length scale corresponding to the film thickness of a waveless flow. The parameter δ was first introduced in [21]. The variables are related with the dimension variables decorated by the upper tilde as:

$$\tilde{t}=\frac{H_c}{\kappa U_c}t,\quad \tilde{x}=\frac{H_c}{\kappa}x,\quad \tilde{h}=H_c h,\quad \tilde{q}=U_c H_c q,\quad \kappa=\left(\frac{2025\delta^2}{\gamma^3}\right)^{1/11},\quad \gamma=\frac{\sigma}{\rho(\nu^4 g)}^{1/3},$$

where a velocity scale $U_c=gH_c^2/3\nu$ is also used.

Two steps are applied in the derivation of the system (1) [14,15]. The first step is to use the thin layer approximation (or the boundary layer approximation for a falling film) including self-induced capillary pressure. This approximation provides the balance between viscous, inertial,